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A MIXED INTEGER NONLINEAR PROGRAMMING MODEL FOR PRODUCT POSITIONING PROBLEM

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ABSTRACT

Product positioning in a market of big city, such as, Medan, Indonesia, is about visibility and recognition and what product represents for a buyer. Consumers usually differ in their choice of an object out of an existing set, and they would also differ if asked to specify an ideal object. The aim of the problem considered is to optimally design a new product in order to attract the largest number of consumers. This paper proposes a mixed integer nonlinear programming model to formulate the problem. A direct search approach based on reduced gradient is proposed to solve the model.

Keywords: *Product positioning problem, modeling, MINLP, reduced gradient, marketing.*

I. INTRODUCTION

There is no product in the world that does not have a position. In marketing point of view, positioning is a crucial element as this is how the general public views a brand or product. The public perception of the brand or product is likely to be a determining factor as to whether they buy it or not In markets where the intensiveness of rivalry and competition are increasing and buyers have a greater intrinsic values become critical. An offering with a clear identity and orientation to needs will not only be purchased, but can warrant a large margin through increased added value.

There is a great number of different definitions of positioning in scientific literature of marketing. In fact, product positioning is defined in the minds of consumers taking into consideration the rest of the market offers. Accordingly, in order to obtain a particular position, consumer product perceptions must be analysed in great detail.

Based on this concept, [30] states that positioning is related with creating brand perceptions in the minds of consumers and with achieving differentiated images apart from competitors' brands/offerings and meeting customer needs/expectations. In other words, positioning of a product depends somehow to consumers' memory of the product brand's particular interesting information given ([31]). A brands' position differentiates it from competitors on attributes considered important by target customers and gives it a distinctive identity in their minds ([32]).

The concept of positioning seeks to place a product in a certain respective buyers. Marketers offerings from those of competitors and to create promotions that communicate the desired position. [20] refer to a product and to differentiate (position) it in a favorable way from similar products. However, [23] defines positioning as the act of designing the upies a distinct and valued position in the target consumers mind. [33] verify that market positioning is arranging for a product to occupy a clear, distinctive, and desirable place, in the minds of target consumers, relative to competing products. Thus, marketers plan positions that distinguish their products from competing products and give them the greatest strategic advantage in their target markets. [34] discuss the need to understand consumer perceptions in order to correctly design product packing and to achieve the desired position in the mind of consumers. According to [29], in developing a positioning strategy, it is necessarily to consider the target market, how the product is different or better than competitors, the value of this difference to the target market and the ability to demonstrate or communicate this difference to the target market . [35] study the relationship between marketing performance and competitive advantage in the commercial banks as a positioning strategy,[31] aimed to understand how the attributes of Chinese tea beverage brands influence consumers' evaluations of brand positioning and differences and competition among brands. The results indicate that, Chinese beverage brands use quenching thirst, attractive advertising and reliable quality aims to develop their positions. [38] presents a literature review supported by an empirical research on the positioning food products in the Romanian market..

Most products and services have many physical and intangible attributes with varied consequences for a would-be purchaser Therefore in product positioning it is necessary to do, what is called perceptual mapping . In terms of this concept, product positioning refers to the positioning of a product in a perceptual attribute space such that it closely to the consumer choice [18]. For a marketing manager, optimizing a new Product's positioning is a critical and difficult decision. Addressing this issue, ([17]) developed a framework for identifying optimal new product concepts using joint space models of consumer perceptions and preferences. Joint space analysis entails mapping the locations of existing products and ideal points for each individual (or market segment) use multidimensional scaling (MDS) of consumer perceptions via factor analysis, discriminate analysis or similarity scaling. Using this joint mapping of ideal points and product locations, a manager can model consumers' choices of existing products, predict their responses to new products, and identify optimal new product concepts. MDS is a series of techniques to support marketer to identify key dimensions relating to consumer's evaluation of a product.

[9] also emphasizes about the important of product positioning in marketing management. [10] discuss about positioning products in which the market has rapid changes in technology and customer preferences. They propose a mathematical model, in which the model aims to maximize the profit from remanufacturing, given a number of units of end-of-life product.

In the ensuing time period, there have been a number of algorithms developed to identify optimal new product positions from MDS-based maps of consumer perceptions and preferences. Thorough reviews of the MDS-based product positioning literature can be found in ([18]) hereafter SMS, ([6], [8]). Each step in this evolution was motivated, in part, by attempts to improve the realism of the consumer choice setting. For example, the algorithms that account for a probabilistic choice model tends to provide better solutions, larger share projections, for new product positions ([16]).

[26] present a unified methodology for product line optimization that coordinates positioning and design model to achieve realizable firm-level optima. [27] propose an ABC curve method for product positioning optimization. Pharmaceutical companies are adopting various positioning strategies. [28] presents a model based on conceptual understanding of various positioning typologies with respect to pharmaceutical companies.

In this paper we assume that the consumer first decides his/her budget for buying from a product class. Then the consumer identifies the set of products from the product class that meet his/her budget constraint, evaluate them with the help of a weighted multi-attribute utility model and chooses the product with the highest utility. Therefore we could propose a mixed integer nonlinear programming (MINLP) model to solve the firm's problem of identifying an optimal new product position. The objective is to identify a point in the multi-dimensional attribute space that is closer than the existing product in the product class to the ideal point of as many consumers as possible.

The organization of this paper is as follows. In the next section, we briefly discuss previous research on MDS-based optimal product positioning and building the 'perfect' product. This is followed by a description of the model. The algorithm and results are presented next. We conclude the paper with a discussion of the result.

II. OPTIMAL POSITIONING LITERATURE REVIEW

In their review, [17] formalized the process of identifying optimal new product concepts using input from consumers at every stage from defining the market to predicting the success of a new product. Since then, a number of algorithms have been developed for MDS-based product positioning. The early approaches ([2], [3], [5]) had two limitations in common. First, the search methods for these procedures were dependent on the number of ideal points (individuals or segments) in the joint space. Consequently, as the number of ideal points rose, so did the complexity of the optimization problem. Second, these algorithms were formulated for the single choice problem in which the demand from each ideal point is assumed to be completely captured by the closest product to it. In essence, this model suggests a consumer always chooses the product nearest to their ideal. While the first limitation simply slowed down the convergence to a suitable solution, the second limitation ignored empirical evidence about the nature of consumers' choices in many consumer markets.

It has been shown in studies of panel data beginning with [12] that consumers often choose probabilistically from a small set of products in the market. One might attribute this behavior to the effects of promotions or availability. However, it has been observed that even if all brands are equally available at no cost, most (53 out of 77) consumers do not choose only their most preferred brand [4]. This indicates that the probabilistic choice behavior may be a product of variety seeking or factors other than environmental effects [14].

In order to introduce a new product it is necessary to have what is called ideal point, defined as consumers desires referring to the attributes consumers would like the products to possess. [18]

presented a product positioning algorithm called PRODSRCH which incorporated a probabilistic model of consumer choice. In their formulation, demand from an ideal point is distributed to a product in inverse proportion its relative distance from the ideal point so long as the product is within the fixed size choice set of the ideal point. Otherwise, the product captures no demand share from that ideal point. [11] address a new methodology for optimal product positioning by considering engineering constraints. The method is based on perceptual mapping and house quality in order to link the consumer perceptual space, and product engineering space.

For multi preference effect of a product, [37] propose a multiple ideal point model. This model uses a product switching matrix.

To illustrate the differences between the single choice model and the probabilistic choice model, we will use the [15] spatial choice model for finite ideal points.

We define

- $x_{i,p}$ is the location ith ideal point on the pth dimension,
- $y_{j,p}$ is the modal perception of the jth product on the pth dimension,
- $w_{i,p}$ is the relative importance of the pth dimension to the ith ideal point,
- S_i is the sales potential for ideal point i.

The weighted Euclidean distance ($d_{i,j}$) between the ith ideal point and jth product position is given by Eq. (1).

$$d_{i,j} = \left(\sum_p w_{i,p} (x_{i,p} - y_{i,p})^2 \right)^{\frac{1}{2}} \qquad (1)$$

In the single choice model, the demand captured by product j is S_i if $d_{i,j} < d_{i,J}$ for all $j \neq J$. In the probabilistic choice model, the share of an ideal point's demand captured by a given product j is determined by the size of the choice set (k) and the relative distances of all available products. It is assumed that due to self interest, consumers are more likely to choose products closest to their ideal points [1].

The brand share for product j from the ith ideal point ($\pi_{i,j}$) is based on Eq. (2):

$$\pi_{i,j} = \begin{cases} \dfrac{(1/d_{i,j})}{\sum_k (1/d_{i,k})} & \text{for the } k \text{ closest products} \\ 0 & \textit{otherwise} \end{cases} \qquad (2)$$

To determine the demand for product j, the share from the ideal point ($\pi_{i,j}$) is multiplied by the sales potential of the ith ideal point (S_i). Another advantage of PRODSRCH is that it relies on a well tested general purpose non-linear programming algorithm known as QRMNEW [13]. Consequently, the complexity of the problem is determined by the number of dimensions of the search space (product dimensions) rather than the number of ideal points and product positions. For MDS-based product positioning, PRODSRCH is considered to be best approach for the single product location problem [6]. [7] proposed a Conjoint Analysis approach for solving the positioning of a product problem.

III. MATEMATICAL STATEMENT OF THE PROBLEM

In this Section we formulate a mathematical programming model for product positioning problem in a multi-attribute space.
Let N be the number of consumers who are a representative sample of the common population for a certain price range of a product class. Else, let M be the number of an existing product (e. g. different brands of cars) in a market which are evaluated by consumers and are located in a multiattribute space of dimension K. We then define

z_{ik} - ideal point on attribute k for the ith consumer, $i = 1, \dots, N; k = 1, \dots, K$

w_{ik} - weight given to attribute k by the ith consumer, $i = 1, \dots, N; k = 1, \dots, K$

δ_{jk} - ideal point on attribute k for the ith consumer, $i = 1, \dots, N; k = 1, \dots, K$

Furthermore, a region (hyper ellipsoid) defining the distance of each consumers to the ideal point can be determined in terms of the existing product, in a way to produce a formulation such that each consumer will select a product which is closest to his/her ideal point. It was mentioned above that the objective of the problem is to optimally design a new product $(x_k, k = 1, \dots, K)$ so as to attract the largest number of consumers.
[39] have extended the scope of the positioning problem by introducing the revenue of the firm from the new product sales to consumer i , C(i) as well as a function f for representing the cost of reaching locations of the new product within an attribute space. Now, the objective of the problem would be to maximize the profits the firm. The binary variable (y_i) is introduced for each consumer to denote whether he/she is attracted by the new product or not.

Consider a positioning problem in which there are 10 existing products (M), 25 consumers (N) and attributes (K). The algebraic representation of such a problem can be written as follows.

$$\text{Maximize } F = \sum_{i=1}^{25} c_i y_i - 0.6x_1^2 + 0.9x_2 + 0.5x_3 - 0.1x_4^2 - x_5$$

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$$\sum_{k=1}^{5} w_{ik}(x_k - z_{ik})^2 - (1 - y_i)H \leq R_i^2, \qquad i = 1, \dots, 25$$

$$x_1 - x_2 + x_3 + x_4 + x_5 \leq 10$$

$$0.6x_1 - 0.9x_2 - 0.5x_3 + 0.1x_4 + x_5 \leq 0.64$$

$$x_1 - x_2 + x_3 - x_4 + x_5 \geq 0.69$$

$$0.157x_1 + 0.05x_2 \leq 1.5$$

$$0.25x_2 + 1.05x_4 - 0.3x_5 \geq 4.5$$

$$2.0 \leq x_1 \leq 4.5$$

$$0.0 \leq x_2 \leq 8.0$$

$$3.0 \leq x_3 \leq 9.0$$

$$0.0 \leq x_4 \leq 5.0$$

$$4.0 \leq x_5 \leq 10.0$$

$$0 \leq y_i \leq 1 \text{ and integer } \forall_i$$

Where

$$R_i^2 = \min_{j=1,\dots,10} \{\sum_{k=1}^{5} w_{ik}(\delta_{jk} - z_{ik})^2\}, \qquad i = 1, \dots, 25$$

$$C^T = [1, 0.2, 1, 0.2, 0.9, 0.9, 0.1, 0.8, 1.0, 0.4, 1, 0.3, 0.1, 0.3, 0.5, 0.9, 0.8, 0.1, 0.9, 1, 1, 1, 0.2, 0.7, 0.7]$$

and H = 1000.

The data for the coordinates of existing product (δ_{jk}), ideal points (z_{ik}) and attribute weights (w_{ik}) can be obtained in [39].
It can be seen that the above formulation is a MINLP model and it contains 25 binary variables, 5 continuous bounded variables, 30 inequality constraints (25 of them acting nonlinearly) and a nonlinear objective function.

IV. THE ALGORITHM

The first four sets of Table 1 partition the full index set , {1, 2, …, n}, ie $J_B \cup J_S \cup J_L \cup J_U = \{1, 2, \dots, n\}$ and $J_\alpha \cap J_\beta = \emptyset$, $\alpha \neq \beta$. The set J_1 of indices corresponding to integer variables is assumed to be of small cardinality, and $m + n_S + n_L + n_U = n$.

The approach assumes that the continuous problem is solved, and seeks an integer-feasible solution in the close neighborhood of the continuous solution. The general philosophy is to leave non-basic integer variables at their respective bounds (and therefore integer valued) and conduct a search in the restricted space of basics, superbasics, and nonbasic continuous variables, $j \notin J_I$.

The algorithm may be broadly summarized as follows:

1. Obtain solution of the continuous relaxation as a nonlinear programming problem.

2. CYCLE1: remove integer variables from the basis by moving a suitable nonbasic away from its bound. The hope is to drive an infeasible integer basic variable to an integer value, and then to pivot it into the superbasic set; the previous nonbasic replacing it in the basis. Some notation is first needed. We define the required index sets.

Table 1. Index set for extended simplex partition.

Name	Meaning	Cardinality
J_B	set of indices for basic variables	$\mid J_B \mid = m$
J_S	set of indices for superbasic variables	$\mid J_S \mid = n_S$
J_L	set of indices for nonbasic variables at their lower bounds	$\mid J_L \mid = n_L$
J_U	set of indices for nonbasic variables at their upper bounds	$\mid J_U \mid = n_U$
J_I	set of indices for integer variables	$\mid J_I \mid = n_I$

3. CYCLE2, pass1: adjust integer-infeasible superbasics by fractional steps to reach complete integer-feasibility.

4. CYCLE2, pass2: adjust integer feasible superbasics. This phase aims to conduct a highly-localized neighborhood search see [36] to verify local optimality.

In Cycle1, there are several steps.

Step 1. Get row i^* the smallest integer infeasibility, such that $\delta_{i^*} = \min\{f_i, 1 - f_i\}$

Step 2. Do a pricing operation

$$v_{i^*}^T = e_{i^*}^T B^{-1}$$

Step 3. Calculate $\sigma_{ij} = v_{i^*}^T a_j$

With j corresponds to $\min_j \left\{ \left| \frac{d_j}{\sigma_{ij}} \right| \right\}$

Calculate the maximum movement of nonbasic j at lower bound and upper bound

Otherwise go to next non-integer nonbasic or superbasic j (if available). Eventually the column j^* is to be increased form LB or decreased from UB. If none go to next i^*.

Step 4. Solve $B\alpha_{j^*} = \alpha_{j^*}$ for α_{j^*}

Step 5. Do ratio test for the basic variables in order to stay feasible due to the releasing of nonbasic j^* from its bounds.

Step 6. Exchange basis

Step 7. If row $i^* = \{\emptyset\}$ go to Stage 2, otherwise

Repeat from step 1.

V. CONCLUSION

Firms work to position and design lines of products that best suit to market and profitability goals. The firms can interpret this imperative by measuring the customer preferences and positioning new products for marketers and maximizing performance under technological constraints. This paper presents a mixed integer nonlinear programming model to describe the positioning of a new product based on multidimensional scaling (MDS) of consumer perceptions.

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