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MATHEMATICAL MODELING OF GROUNDWATER FLOW

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ABSTRACT

The aim of this paper is to design the mathematical model of groundwater flow. Groundwater is not static, it flows in an aquifer and its flow can be described using partial differential equation and associated initial-boundary conditions. The work considered three dimensional steady state groundwater flows. Then the model of groundwater flow in discharging an aquifer at a well and the flow of water under a dam solved using the Laplace homotopy perturbation method, the paper focused on groundwater flow under a dam and in discharging an aquifer at a well.

Keywords: *Mathematical model, Aquifers, stream-aquifer interaction, Dupuit assumptions, Partial differential equation, Laplace transform, Homotopy perturbation method..*

I. INTRODUCTION

In the past, the main driving force for hydrogeologic studies has been the need to assess the water-supply potential of aquifers. During the past 20 years, however, the emphasis has shifted from water-supply problems to water-quality problems. This has driven a need to predict the movement of contaminants through the subsurface environment. One consequence of the change in emphasis has been a shift in perceived priorities for scientific research and data collection. Formerly, the focus was on developing methods to assess and measure the water-yielding properties of high-permeability aquifers. The focus is now largely on transport and dispersion processes, retardation and degradation of chemical contaminants, the effects of heterogeneity on flow paths and travel times.

The past 20 years or so have also seen some major technological breakthroughs in groundwater hydrology. One technological growth area has been in the development and use of deterministic, distributed-parameter, computer simulation models for analysing flow and solute transport in groundwater systems. These developments have somewhat paralleled the development and widespread availability of faster, larger memory, more capable, yet less expensive computer systems. Another major technological growth area has been in the application of isotopic analyses to groundwater hydrology, wherein isotopic measurements are being used to help interpret and define groundwater flow paths, ages, leakage, and interactions with surface water [1-4].

II. MODELS

The word model has so many definitions and is so overused that it is sometimes difficult to discern the meaning of the word (Konikow and Bredehoeft, 1992 in [5]). A model is perhaps most simply defined as a representation of a real system or process. A conceptual model is a hypothesis for how a system or process operates. This hypothesis can be expressed quantitatively as a mathematical model. Mathematical models are abstractions that represent processes as equations, physical properties as constants or coefficient in the equations, and measures of state or potential in the system as variables.

Most groundwater models in use today are deterministic mathematical models. Deterministic models are based on conservation of mass, momentum, and energy and describe cause and effect relations. The underlying assumption is that given a high degree of understanding of the processes by which stresses on a system produce subsequent responses in that system, the system's response to any set of stresses can be predetermined, even if the magnitude of the new stresses falls outside of the range of historically observed stresses.Deterministic groundwater models generally require the solution of partial differential equations. Exact solutions can often be obtained analytically, but analytical models requirethat the parameters and boundaries be highly idealised. Some deterministic models treat the properties of porous media as lumped parameters (essentially, as a black box), but this precludes the representation of heterogeneous hydraulic properties in the model. Heterogeneity, or variability in aquifer properties, is characteristic of all geologic systems and is now recognised as playing a key role in influencing groundwater flow and solute transport. Thus, it is often preferable to apply distributed-parameter models, which allow the

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representation of more realistic distributions of system properties. Numerical methods yield approximate solutions to the governing equation (or equations) through the discretisation of space and time. Within the discretised problem domain, the variable internal properties, boundaries, and stresses of the system are approximated. Deterministic, distributed-parameter, numerical models can relax the rigid idealised conditions of analytical models or lumped-parameter models [6-10].

The number and types of equations to be solved are determined by the concepts of the dominant governing processes. The coefficients of the equations are the parameters that are measures of the properties, boundaries, and stresses of the system; the dependent variables of the equations are the measures of the state of the system and are mathematically determined by the solution of the equations. When a numerical algorithm is implemented in a computer code to solve one or more partial differential equations, the resulting computer code can be considered a generic model. When the grid dimensions, boundary conditions, and other parameters (such as hydraulic conductivity and storativity), are specified in an application of a generic model to represent a particular geographical area, the resulting computer program is a site-specific model. The ability of generic models to solve the governing equations accurately is typically demonstrated by example applications to simplified problems [11-12].

III. DESIGN OF GROUNDWATER FLOW MODEL EQUATION

The standard equations that govern groundwater flow are derived using the principle of continuity and Darcy's law. Consider a three dimensional prototype volume element of a porous soil medium abstracted from an aquifer, as shown in the Figure 1

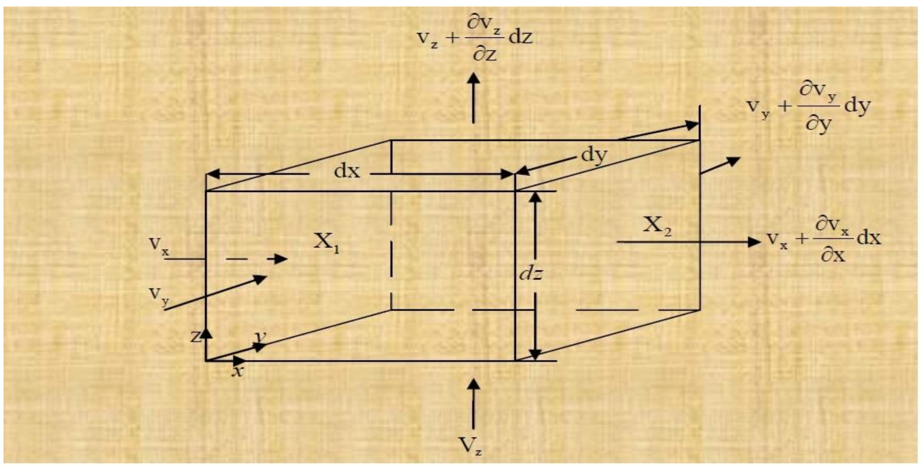

Figure 1: A Typical Volume Element in a Porous Aquifer

The magnitude of the water velocity across each surface of the representative volume element changes with time. Typically, if the velocity at the entry surface X_1 is v_x, then the velocity at the exit surface X_2 becomes:

$$v_x + \frac{\partial v_x}{\partial x} dx \qquad (1)$$

The net balance of flow within the representative volume element has four contributing components. Three components come from the net flows in the *x*,*y* and *z*directions. The fourth component is due to the compressibility of the aquifer system, and is equal to the specific storage, abbreviated S_0, multiplied by the rate of change of the groundwater head with respect to time. The sum of all four components is zero. Specifically, for the flow in the *x*-direction we have:

Inflow in time δt equals $v_x dydz\delta t$

Outflow in time δt equals $\left(v_x + \frac{\partial v_x}{\partial x} dx\right) dy\, dz\, \delta t$

Net balance of flow in time δt equals inflow minus outflow

$v_x dydz\delta t$ - $\left(v_x + \frac{\partial v_x}{\partial x}dx\right) dy\ dz\ \delta t\ = -\left(\frac{\partial v_x}{\partial x}dx\right) dy\ dz\ \delta t$

Similarly one shows that the net balance of flow in *y* and *z* directions are;

$v_y dx\ dz\ \delta t$ - $\left(v_y + \frac{\partial v_y}{\partial y}dy\right) dx\ dz\ \delta t\ = -\left(\frac{\partial v_y}{\partial y}dy\right) dx\ dz\ \delta t$

$v_z dx\ dy\ \delta t$ - $\left(v_z + \frac{\partial v_z}{\partial z}dz\right) dx\ dy\ \delta t\ = -\left(\frac{\partial v_z}{\partial z}dz\right) dx\ dy\ \delta t$

The net flow in time δt due to increase in head equals $-\delta h\ S_s dx\ dy\ dz$
The sum of all four components is zero. Therefore

$$-\left(\frac{\partial v_x}{\partial x}dx\right) dy\ dz\ \delta t\ -\left(\frac{\partial v_y}{\partial y}dy\right) dx\ dz\ \delta t\ -\left(\frac{\partial v_z}{\partial z}dz\right) dx\ dy\ \delta t\ -\ \delta h S_0 dx\ dy\ dz = 0$$

By expressing the term $-\delta h\ S_0 dx\ dy\ dz$ in the form $-\frac{\delta \mathrm{h}}{\delta t}\mathrm{S}_s\ dx\ dy\ dz\ \delta t$ the above equation becomes

$$-\left(\frac{\partial v_x}{\partial x}dx\right) dy\ dz\ \delta t\ -\left(\frac{\partial v_y}{\partial y}dy\right) dx\ dz\ \delta t\ -\left(\frac{\partial v_z}{\partial z}dz\right) dx\ dy\ \delta t\ -\ \frac{\delta h}{\delta t}S_0\ dx\ dy\ dz\ \delta t = 0$$

Deletion of the common factor $dx\ dy\ dz\ \delta t$ in the equation and noting that in the limit, $\frac{\delta h}{\delta t}$ tends to $\frac{\partial h}{\partial t}$ finally leads to the equation :

$$\left(\frac{\partial v_x}{\partial x}\right) + \left(\frac{\partial v_y}{\partial y}\right) + \left(\frac{\partial v_z}{\partial z}\right) + \frac{\partial h}{\partial t}S_0 = 0 \qquad (2)$$

But Darcy's law stipulates that the velocities of the hydraulic head in the *x*, *y* and *z* directions are, respectively,

$v_x = -K_x \frac{\partial h}{\partial x}$, $v_y = -K_y \frac{\partial h}{\partial y}$, and $v_z = -K_z \frac{\partial h}{\partial z}$ (3)

Substitution of these velocities into equation (2), and the fact that K_x, K_y and K_z are constants, results in the equation

$$\left(K_x \frac{\partial^2 h}{\partial x^2}\right) + \left(K_y \frac{\partial^2 h}{\partial y^2}\right) + \left(K_z \frac{\partial^2 h}{\partial z^2}\right) = S_0 \frac{\partial h}{\partial t} \qquad (4)$$

This is the general equation for three dimensional groundwater flow.
Since T=Km and S=S_0m then (4) becomes

$$\left(T_x \frac{\partial^2 h}{\partial x^2}\right) + \left(T_y \frac{\partial^2 h}{\partial y^2}\right) + \left(T_z \frac{\partial^2 h}{\partial z^2}\right) = S \frac{\partial h}{\partial t} \qquad (5)$$

Sometime there is a source of ground water in the aquifer, such as a well. Such a source introduces a new term Q(*x*,*y*,*z*) into equation (5) giving

$$\left(T_x \frac{\partial^2 h}{\partial x^2}\right) + \left(T_y \frac{\partial^2 h}{\partial y^2}\right) + \left(T_z \frac{\partial^2 h}{\partial z^2}\right) + Q = S \frac{\partial h}{\partial t} \qquad (6)$$

Where;
K_i: hydraulic conductivity in the i-direction (L/T or m s^{-1}))

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T_i : transmissivity in the i-direction (L^2/T)
S : storage coefficient (-)
m : thickness of aquifer (L)
S_0: specific storage coefficient (1/L)
h: hydraulic head or simply "head" or pressure head (also known as piezometric head) (L)
v_i: velocity in the i-direction(L/T)
Q : the volumetric source per unit volume (discharge rate) (L^3/T).
).

A) In the Steady State Case

In this case the flow is assumed to be under steady state conditions. This implies $\frac{\partial h}{\partial t} = 0$. In this case the governing equations (5) changes from parabolic to elliptic, specifically becoming the Laplace equation .

$$\left(T_x \frac{\partial^2 h}{\partial x^2}\right) + \left(T_y \frac{\partial^2 h}{\partial y^2}\right) + \left(T_z \frac{\partial^2 h}{\partial z^2}\right) = 0 \qquad (7)$$

And equation (6) becomes

$$\left(T_x \frac{\partial^2 h}{\partial x^2}\right) + \left(T_y \frac{\partial^2 h}{\partial y^2}\right) + \left(T_z \frac{\partial^2 h}{\partial z^2}\right) + Q = 0 \qquad (8)$$

In homogeneous soil permeability be equal for all directions, i.e. $T_x=T_y=T_z=T$, therefore the governing equations (5), (6), (7) and (8) become respectively:

$$\left(\frac{\partial^2 h}{\partial x^2}\right) + \left(\frac{\partial^2 h}{\partial y^2}\right) + \left(\frac{\partial^2 h}{\partial z^2}\right) = \frac{S}{T}\frac{\partial h}{\partial t} \qquad (9)$$

$$\left(\frac{\partial^2 h}{\partial x^2}\right) + \left(\frac{\partial^2 h}{\partial y^2}\right) + \left(\frac{\partial^2 h}{\partial z^2}\right) + \frac{Q}{T} = \frac{S}{T}\frac{\partial h}{\partial t} \qquad (10)$$

$$\left(\frac{\partial^2 h}{\partial x^2}\right) + \left(\frac{\partial^2 h}{\partial y^2}\right) + \left(\frac{\partial^2 h}{\partial z^2}\right) = 0 \qquad (11)$$

$$\left(\frac{\partial^2 h}{\partial x^2}\right) + \left(\frac{\partial^2 h}{\partial y^2}\right) + \left(\frac{\partial^2 h}{\partial z^2}\right) + \frac{Q}{T} = 0 \qquad (12)$$

IV. SOLVING SUGGESTED DESIGN

Proceeding in same manner, the rest of the components $h_n(x, y, z, t)$ can be completely obtained and the series solution is thus entirely determined. Finally, we approximate the analytical solution $h(x, y, z, t)$ by truncated series: In this section we solve the suggested design for govern groundwater flow model equation, now rewrite equation (9) as follow:

$$\left(T_x \frac{\partial^2 h}{\partial x^2}\right) + \left(T_y \frac{\partial^2 h}{\partial y^2}\right) + \left(T_z \frac{\partial^2 h}{\partial z^2}\right) = S\frac{\partial h}{\partial t}, \quad x, y, z \in R \text{ and } t > 0 \; (13)$$

With initial condition (IC): h(x,y,z,0) = f(x,y,z)
Where;

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T_i : transmissivity in the i-direction (L^2/T)
S : storage coefficient
h : hydraulic head or simply "head" or pressure head (also known as piezometric head) (L)
now, we solve equation (13) usingLaplaceHomotopy perturbation method (LHPM)as follow:

Taking Laplace transform on both sides of the equation (13) and using the linearity of the Laplace transform

$$T_x\ L\left\{\frac{\partial^2 \mathrm{h}}{\partial x^2}\right\} + T_y\ L\ \left\{\frac{\partial^2 \mathrm{h}}{\partial y^2}\right\} + T_z\ L\ \left\{\frac{\partial^2 \mathrm{h}}{\partial z^2}\right\} = L\left\{S\frac{\partial \mathrm{h}}{\partial t}\right\}(14)$$

By applying the Laplace transform differentiation property, we have

$$sSL\{h(x,y,z,t)\} - Sh(x,y,z,0) = \ T_x\ L\left\{\frac{\partial^2 \mathrm{h}}{\partial x^2}\right\} + T_y\ L\ \left\{\frac{\partial^2 \mathrm{h}}{\partial y^2}\right\} + T_z\ L\ \left\{\frac{\partial^2 \mathrm{h}}{\partial z^2}\right\}(15)$$

Then we have

$$L\{h(x,y,z,t)\} = \frac{f(x,y,z)}{s} + \frac{1}{sS}\left[T_x\ L\left\{\frac{\partial^2 \mathrm{h}}{\partial x^2}\right\} + T_y\ L\ \left\{\frac{\partial^2 \mathrm{h}}{\partial y^2}\right\} + T_z\ L\ \left\{\frac{\partial^2 \mathrm{h}}{\partial z^2}\right\}\right](16)$$

Taking the inverse Laplace transform on equation (16), we get:

$$h = L^{-1}\left\{\frac{f}{s}\right\} + \frac{1}{S}L^{-1}\left[T_x\left\{\frac{1}{s}L\left\{\frac{\partial^2 \mathrm{h}}{\partial x^2}\right\}\right\} + T_y\left\{\frac{1}{s}L\left\{\frac{\partial^2 \mathrm{h}}{\partial y^2}\right\}\right\} + T_z\left\{\frac{1}{s}L\left\{\frac{\partial^2 \mathrm{h}}{\partial z^2}\right\}\right\}\right](17)$$

In the homotopy perturbation method (HPM), the basic assumption is that the solutions can be written as a power series in *p* such:

$$h(x,y,z,t) = \sum_{n=0}^{\infty} p^n h_n(x,y,z,t)(18)$$

Where$p \in [0,1]$ is an embedding parameter.
Substituting (18) in (17), we get:

$$\sum_{n=0}^{\infty} p^n h_n = L^{-1}\left\{\frac{f}{s}\right\}$$
$$+\frac{1}{S}pL^{-1}\left[\frac{T_x}{s}L\left\{\sum_{n=0}^{\infty} p^n\frac{\partial^2 \mathrm{h}_n}{\partial x^2}\right\} + \frac{T_y}{s}L\left\{\sum_{n=0}^{\infty} p^n\frac{\partial^2 h_n}{\partial y^2}\right\} + \frac{T_z}{s}L\left\{\sum_{n=0}^{\infty} p^n\frac{\partial^2 h_n}{\partial z^2}\right\}\right](19)$$

This is the coupling of the Laplace transform and the homotopy perturbation method. Comparingthe coefficient of like powers of *p*, the following approximations are obtained:

$$\left.\begin{aligned}
&p^0: \quad h_0(x,y,z,t) &&= L^{-1}\left\{\frac{f}{s}\right\} \\
&p^1: \quad h_1(x,y,z,t) &&= \frac{1}{S}L^{-1}\left\{\frac{T_x}{s}L\left\{\frac{\partial^2 h_0}{\partial x^2}\right\}+\frac{T_y}{s}L\left\{\frac{\partial^2 h_0}{\partial y^2}\right\}+\frac{T_z}{s}L\left\{\frac{\partial^2 h_0}{\partial z^2}\right\}\right\} \\
&p^2: \quad h_2(x,y,z,t) &&= \frac{1}{S}L^{-1}\left\{\frac{T_x}{s}L\left\{\frac{\partial^2 h_1}{\partial x^2}\right\}+\frac{T_y}{s}L\left\{\frac{\partial^2 h_1}{\partial y^2}\right\}+\frac{T_z}{s}L\left\{\frac{\partial^2 h_1}{\partial z^2}\right\}\right\} \\
&\vdots \\
&\text{... and so on.}
\end{aligned}\right\} \quad (20)$$

Proceeding in same manner, the rest of the components $h_n(x, y, z, t)$ can be completely obtained and the series solutionis thus entirely determined. Finally, we approximate the analytical solution $h(x, y, z, t)$ by truncated series:

$$h(x,y,z,t) = \lim_{N\to\infty}\sum_{n=0}^{N} h_n(x,y,z,t)(21)$$

V. APPLICATIONS

Now we applied the above study for some important cases as follow

Problem 1

Suppose that $h(x,y,z,0)=f(x,y,z)=ax^2+by^2+cz^2+h_0$, where a,b,c and h_0 are constants.
Then by equation (20) we get the powers of p as following:

P^0 :$h_0(x,y,z,t)=ax$^2+ by^2+ cz^2+ h_0

P^1 :$h_1(x,y,z,t)=(2t / S)(a\, Tx+ b\, Ty + c\, Tz)$

P^2 : $h_2(x,y,z,t)=0$

P^3 : $h_3(x,y,z,t)=0$

$\vdots$

Then the solution is

$$\therefore h(x,y,z,t) = \lim_{N\to\infty}\sum_{n=0}^{N} h_n(x,y,z,t) = ax^2+by^2+cz^2+h_0+(\frac{2t}{S})(aT_x+bT_y+cT_z)$$

Problem 2

Suppose that $h(x,y,z,0)=f(x,y,z)=d\sin(ax)\sin(by)\sin(cz)+h_0$, where a,b,c,d and h_0 are constants.
Then by equation (20) we get the powers of p as following:

P^0 :$h_0(x,y,z,t) = d\sin(ax)\sin(by)\sin(cz)+h_0$

P^1 : $h_1(x,y,z,t)$ = - $(d\sin(ax)\sin(by)\sin(cz))\,(t / S)(a^2Tx+ b^2\, Ty + c^2Tz)$

P^2 : $h_2(x,y,z,t)$ = $(d\sin(ax)\sin(by)\sin(cz))\,(t^2 / 2S^2)\,(a^2Tx+ b^2\, Ty + c^2Tz)^2$

P^3 : $h_3(x,y,z,t)$ = $-(d\sin(ax)\sin(by)\sin(cz))\,(t^3 / 3!\,S^3)\,(a^2Tx+ b^2\, Ty + c^2Tz)^3$

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$P^4 : h_4(x,y,z,t) = (d \sin(ax) \sin(by) \sin(cz)) (t^4 / 4!S^4) (a^2Tx + b^2 Ty + c^2Tz)^4$

$P^5 : h_5(x,y,z,t) = -(d \sin(ax) \sin(by) \sin(cz)) (t^5 / 5!S^5) (a^2Tx + b^2 Ty + c^2Tz)^5$

⋮

$P^n : h_n(x,y,z,t) = (-1)^n (d \sin(ax) \sin(by) \sin(cz)) (t^n / n!S^n) (a^2Tx + b^2 Ty + c^2Tz)^n$

Then the solution is

$$\begin{aligned} h(x,y,z,t) &= \lim_{N\to\infty} \sum_{n=0}^{N} h_n(x,y,z,t) \\ &= \sum_{n=0}^{\infty} h_n(x,y,z,t) \\ &= h_0 + \sum_{n=0}^{\infty} (-1)^n d \sin(ax) \sin(by) \sin(cz) \left(\frac{t^n}{n!\ S^n}\right) (a^2T_x + b^2T_y + c^2T_z)^n \end{aligned}$$

$$\therefore h(x,y,z,t) = h_0 + d \sin(ax) \sin(by) \sin(cz)\, e^{-\frac{t}{S}(a^2T_x + b^2T_y + c^2T_z)}$$

Problem 3

Suppose that $h(x,y,z,0)=f(x,y,z)=\lambda \sin(ax)+ \beta \sin(by)+ \alpha \sin(cz) + h_0$, where $a,b,c,d,\lambda,\beta,\alpha$ and h_0 are constants.
Then by equation (20) we get the powers of p as following:

$P^0 : h_0(x,y,z,t) = \lambda \sin(ax)+ \beta \sin(by)+ \alpha \sin(cz) + h_0$

$P^1 : h_1(x,y,z,t) = - (\lambda a^2Tx \sin(ax)+ \beta b^2 Ty \sin(by)+ \alpha c^2Tz\sin(cz)) (t / S)$

$P^2 : h_2(x,y,z,t) = (\lambda a^4(Tx)^2 \sin(ax)+ \beta b^4(Ty)^2 \sin(by)+ \alpha c^4(Tz)^2\sin(cz)) (t^2 / 2S^2)$

$P^3 : h_3(x,y,z,t) = - (\lambda a^6(Tx)^3 \sin(ax)+ \beta b^6(Ty)^3 \sin(by)+ \alpha c^6(Tz)^3\sin(cz)) (t^3 / 3!\ S^3)$

$P^4 : h_4(x,y,z,t) = - (\lambda a^8(Tx)^4 \sin(ax)+ \beta b^8(Ty)^4 \sin(by)+ \alpha c^8(Tz)^4\sin(cz)) (t^4 / 4!\ S^4)$

$P^5 : h_5(x,y,z,t) = - (\lambda a^{10}(Tx)^5 \sin(ax)+ \beta b^{10}(Ty)^5 \sin(by)+ \alpha c^{10}(Tz)^5\sin(cz)) (t^5 / 5!\ S^5)$

⋮

$P^n : h_n(x,y,z,t) = (-1)^n((\lambda a^{2n}(Tx)^n \sin(ax)+ \beta b^{2n}(Ty)^n \sin(by)+ \alpha c^{2n}(Tz)^n\sin(cz))) (t^n / n!\ S^n)$

Then the solution is

$$h(x,y,z,t) = \lim_{N\to\infty} \sum_{n=0}^{N} h_n(x,y,z,t) = \sum_{n=0}^{\infty} h_n(x,y,z,t)$$

$$= h_0 + \sum_{n=0}^{\infty} (-1)^n \lambda\, a^{2n} (T_x)^n \sin(ax) \left(\frac{t^n}{n!\ S^n}\right)$$
$$+ \sum_{n=0}^{\infty} (-1)^n \beta\, b^{2n} (T_y)^n \sin(by) \left(\frac{t^n}{n!\ S^n}\right)$$
$$+ \sum_{n=0}^{\infty} (-1)^n \alpha\, c^{2n} (T_z)^n \sin(cz) \left(\frac{t^n}{n!\ S^n}\right)$$

$$\therefore h(x,y,z,t) = h_0 + \lambda \sin(ax)\, e^{-\frac{t}{S}(a^2 T_x)} + \beta \sin(by)\, e^{-\frac{t}{S}(b^2 T_y)} + \alpha \sin(cz)\, e^{-\frac{t}{S}(c^2 T_z)}$$

VI. CONCLUSION

The approximate analytical solution of groundwater flow equation for aquifer is designed and solved by applying LHPM. We see that LHPM is efficient, accurate and convenient. The combination of Homotopy perturbation method and Laplace transform overcomes the restriction of Laplace transform method to solve non-linear partial differential equation. The two important parameters viz. Hydraulic conductivity and specific yield S are considered in the present groundwater flow problem. The approximate analytical solution is obtained.

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