[Azimi, 4(7): July 2017] ISSN 2348 – 8034
DOI- 10.5281/zenodo.825025 Impact Factor- 4.022

GLOBAL JOURNAL OF ENGINEERING SCIENCE AND RESEARCHES

DEVELOPING A MODEL BASED ON IMAGE PROCESSING FOR SOIL SLOPE STABILITY ASSESSMENT

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ABSTRACT

Slope stability is one of the most sophisticated and challenging problems for all mining and geotechnical engineeringdisciplines. In recent decades, substantial progress in the knowledge and practice concerning slope analysis has been made. Surface failure is both a natural and artificial phenomenon, with littleinformation about the mechanismsbehind suchfailure. The importance of slope stability analysis has increased because of a significant increase in the frequency and consequence of natural hazards. Therehas been a gradual increase in the power of computers and significant progress has been made in the development of specialised software based on various methods ofslope stability analysis under dynamic and static conditions. Based on the importance of uncertainty in slope stability analysis, an increasing role for image processing techniques is ensured. Image processing techniques are crucial for both regional and site-specific studies focusing on slope stability analysis. The presentstudyemploys one of the most popular image processing techniques, partial image velocimetry, to undertake slope stability investigations.

Keywords*: particle image velocimetry, slope stability analysis, image processing, uncertainty*.

I. INTRODUCTION

From the beginning of civilisation, humanshave deliberately attempted to discover more stable areas for their domiciles. With rapid expansion of population sizeand industrialisation, a sharp increase in establishing infrastructure, such as railways, highways, and roads, has occurred. To facilitate such developments, human-made cut and fill slopes are ineluctable during the construction process. In the majority of cases, the slopes strongly facilitate the convenience of humans, whereasin other situations, past disasters related to geotechnological problems have been closelyrelated to slope instability.

Soil landslides are known as one of the most common geohazard phenomena in the world, particularly in zones with residual soils that have high potential for sliding. This phenomenon is a crucial problem worldwide because it might damage houses, jeopardise continuity of mining activities, destroy roads, prevent road development projects from continuing, and even harm people and/or lead to loss of life. To protect lives and property, it is necessary to discoverthe affected area resulting from the failures (Chen et al., 2011). Furthermore, it can effectively help authorities to accomplish prevention measures and mitigate risk of failure or establishearly warning systems.

However, the slope stability models often formulate a failure phenomenon with low accuracy or high vagueness because of the complexity of factors influencing the soil slope stability; therefore, conventional models neglect to apply all available information. Consequently, they inaccurately estimate the factor of safety(FOS)because of limited information. The merit of using the particle image velocimetry (PIV) technique is that it formulates failure by utilising the exact optical features of the soil slope instead of using analytical and mathematical equations. An initial review of the applications of the PIV technique wasinvestigated by Adrian (1991), which focused on velocimetry measurements in the field of fluid mechanics.

The main aim of the present studywas to investigate the capability of an image processing technique, namely PIV, in the characterisation of the behaviour of infinite soil slopes and evaluation of their stability. The soil samples included in the study were loose sand having varying clay content that were utilised to investigate the effects of fine particles. As such, different compositions including without bentonite, 5% bentonite, 10% bentonite, 15% bentonite and 20% bentonite were examined. The main aim of the research was to immediately and effectively detect and trace the threshold and shape of failures to reduce their adverse effectson slope movements.

II. LITERATURE REVIEW

Slope stability analysis is an important but risky component of civil and mining engineering. The experience of the failure surfaces can teach important lessons to not only understand the causes of failure but also comprehensively evaluate slope stability. Therefore, methods with high capabilities in modelling sophisticated systemshave been developed. Inrecent decades, a gradual increase in the power of computers has occurred and significant progress in the development of technical analysis software based on both conventional and modern techniques of slope stability analysis has taken place. Nonetheless, the selection of the proper technique for slope stability assessment is crucial. To achieve this aim, a set of failure observations and field studies is required to determine the failure mechanisms to select the most appropriate technique for the analysis.

1. Limit Analysis

For a precise result, both the equilibrium and compatibility of the slope need to be taken into accountsimultaneously. This means that the constitutive equations of the material, strain compatibility equations, differential equations of equilibrium and boundary conditions of the problem under consideration are required. The limit analysis models a slope stability problem by utilising the conceptof the stress-strain relationship based on the assumption that the soil is a rigid structure. Without handling a systematic elasto-plastic analysis, many problems can be solved using this analysis (Cheng & Lau, 2008) and the bound formulas of traditional plasticity concept are applied (Drucker et al., 1951).

2. Limit Equilibrium Method (LEM)

The most general method in slope stability analysis is the LEM (Cheng & Lau, 2008). Based on the primary concept of the LEM, a division process of the sliding mass into sections is required to compute a safety factor for a specific failure, which is known as the method of slices. To compute the driving or resisting force or moment, equilibrium conditions are performed for each slice. The summation of the values pertaining to all resisting and driving moments and forces are calculated for the computation of the total FOS.

Swedish Circle Method (SCM)

The SCM was established on the assumption that the failure surface is circular and the FOS is calculated by summing the moments about the centre of the circle. This method, which was first applied by Petterson in 1916 and formalised by Fellenius in 1992 (Duncan et al., 2014), assumes that the friction angle is equal to zero. Otherwise, the shear strength, mathematically shown as the following equation, only results from cohesion ($\phi = 0 \rightarrow \tau = c_d$):

$$\tau = c_d + \sigma \tan \phi_d \quad (1)$$

where,

$$\tan \phi_d = \frac{\tan \phi}{F} \quad (2)$$

The variables c_d and ϕ_d represent the developed cohesion and friction angle, respectively; c and ϕ are the cohesion and friction angle for the soil, respectively; and σ is the total normal stress on the shear plane. The SCM is an appropriate tool for analysing short-term stability of both homogeneous and heterogeneous slopes (Duncan et al., 2014).

Infinite Slope Failures

Shallow landslides are a type of slope failure that might be induced by rainfall infiltration, which causes changes in total suction (i.e. the sum of matric and osmotic suctions) and soil properties such as soil shear strength and pore fluid (e.g. air, water, and dissolved air in water and menisci) properties. The mechanism of change (from unsaturation to saturation and back to unsaturation) for this process is still not clearly understood.

ISSN 2348 – 8034
Impact Factor- 4.022

Methods of Slices

For the methodsof slices, a number of solution techniques have been developed. The primary differences among all the techniques arise from the equations of the considered statics, the included interslice normal and shear forces, and the presumed correlation between the inner slice forces (Krahn, 2003). A typical slice in a potential sliding mass with the forces acting on the slice is depicted in Figure 1. The real number of slices utilised is based on the geometry and profile of the slope earthfill (Duncan et al., 2014).

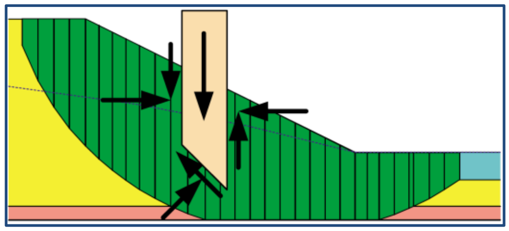

Figure 1: Slices and forces in a sliding mass

A number of the methods of slices formulate a slope stability problem focused on the assumption that the slip surface is a circular slip surface; whereas, a noncircular slip surface is assumed by others. The former considers the equilibrium of moments about the centre of the circle, while the latter considers the equilibrium in terms of the individual slices(Duncan et al., 2014).

The Ordinary Method of Slices

This approach is also known as the Fellenius method or Swedish method of slices. This method is a procedure of slices that neglects the forces on the sides of the slices (Figure 2). As shown in Figure 2(a), the failure surface is divided into a number of imaginary upright slices. In this method, the computational method for calculating the FOSissimple and straightforward according to:

$$F = \frac{\sum\{c' \times l + \tan\varphi'(W \times \cos(\alpha) - u \times l\}}{\sum W \times \sin(\alpha)} \quad (3)$$

where, c' is the effective cohesion; l denotes the slice base length; φ' is the effective friction angle; W denotes the weight of the slice; u expresses the pore water pressure;and α denotes the angle between the tangent of the centreof the base of the slice and thehorizon (Fellenius, 1936).

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[Azimi, 4(7): July 2017]
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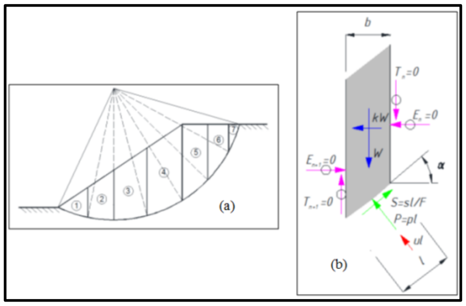

Figure 2:Ordinary method of slices (a) different slices (b) forces acting on single slice (Fellenius, 1936)

Simplified Bishop Method

The simplified Bishop method, proposed originally by Bishop (1955), is based on the statement that the forces on the sides of the slice are flat.Therefore, it is assumed that no shear stresses exist between slices.

3. Noncircular methods

Janbu's Simplified Method

Janbu's simplified method (1954) only considers total horizontal forceequilibrium, instead of total moment equilibrium. This method is similar to the Bishop method in that it only takes into account two of the three equations of the equilibrium problem. Force equilibrium is sensitive to the forces on the sides of the slice; therefore,Janbu's simplified method thatignores these forces gives a less accurate result for circular slip surfaces.

Spencer's (1967) Method

Spencer's (1967) method, which was first developed for circular surfaces, takes into account all conditions of equilibrium, including moment, vertical and horizontal force equilibrium. This method extends to model noncircular surfaces and assumes that all forces on the sides of the slice are parallel.

Morgenstern and Price's (1965) Method

InMorgenstern and Price's (1965) method, all normal, tangential and moment equilibrium are considered for each slice in the circular and noncircular slip surfaces.This method generates two FOS, similar to Spencer's (1967) method, based on moment and horizontal force equilibrium. Since the Morgenstern and Price's (1965) method takes into account force and moment equilibrium, as well as the forces on the sides of the slice, the results are more robust for slope stability analysis.

4. Numerical Modelling

When using numerical modelling, different equations and conditions can be considered during problem formulation, including strain compatibility equations, constitutive equations for material, differential equations of equilibrium and boundary conditions of the problem under consideration(Krahn, 2003). One of the main advantages of numerical modelling is that it computes both the displacement and the stress resulting from external loads. For the formulation of complex slope geometry, numerical modelling can provide a better result compared to analytical models or LEMs(Bobet, 2010).

Finite Element Method (FEM)

Frequently, conventional methods cannot estimate the progressive failure phenomenon. To overcome this limitation, the FEMhas been proposed, with two key applications developed for analysing slope stability.

Distinct Element Method (DEM)

The DEMis capable of providing a qualitative assessment to estimate the complete failure mechanism (Cheng & Lau, 2008). This method employs an approach based on a set of triangular rigid blocks or particles.

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Rigid Element Method (REM)

The REM, also known as the rigid body-spring model proposed by Kawai (1978), was initially developed from the DEM(Cundall, 1971). This technique is also known as the interface element method and rigid finite element method. Although the process of discovering a solution for a problem in the system of the REM is similar to that in the traditional FEM, the REM employs elements and interfaces rather than the nodes and elements ofthe FEM.

5. Empirical Design

To analyse a simple homogeneous slope, the FOS can be extracted from a stability table or figure without using a computational process or computer software. An early attempt was made by Lutton (1970) to develop an organised classification of empirical data. According to the high potential of stability figures and tables in slope stability, several types of figures and tables have been proposed. Generally, stability tables and figures obtain results that are sufficiently close to each other. However, these tables and figures are usually developed for analysing 2D problems.

6. Physical Model Tests

Physical model tests have been developed for solving the problems and costs pertaining to full-scale testing. These tests simulate the conditions of a real slope under a controlled environment, where the influence factors can simply vary and the corresponding impacts on slope stability are comprehensively analysed. The method provides an opportunity for studying the effects of unknown parameters that might be time-consuming and bothersome in the field(Springman et al, 2010).

7. Probabilistic Methods

The basic motivation for development of probabilistic approaches for slope stability analysis is to recognise the variation of factor weights based on natural variations. However, probabilistic methods use distribution functions that require a large amount of input data and assumptions. These methods are based on the LEM and have the same limitationsthatthe LEM does. Likewise, practical applications of probabilistic methods are limited because of the vast amount of input data required to run such methods. However, increasing the amount of required data leads to increasing the rendering time.

8. Image Processing Methods

Image processing is the processing of images employing mathematical methods to fulfil a set of operations on a specific image. These operations are performed for the purpose of extracting valuable information or obtaining a reduced or enhanced image. Image processing is a typical form of signal processing whereby the input is an image (picture), a video or a series of images and the output can be either an image or features (characteristics) pertaining to the image.

Figure 3 depicts that the ray emitted by an object can be influenced by environmental radiations. By refraction of the ray produced by the object, the position of the object might be changed. As shown in the figure, the scattering and absorption processes can lead to attenuation of the radiation fluctuations. As a result, the observations can be falsified. To obtain an accurate result, additional effects should be minimised (Jähne, 2005).

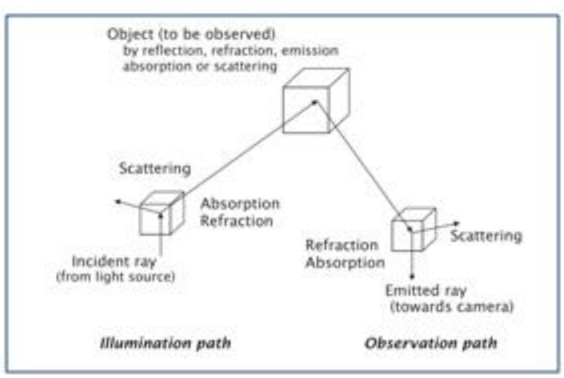

Figure 3: The interaction between the radiative energy emitted by the object and environment rays(Jähne, 2005)

While techniques of flow visualisation have been used from the beginning of hydrodynamics, measurable analysis of image sequences to determine flow parameters has only become available recently (Jähne, 2004). Particle image velocimetry (PIV) is the standard technique for analysis of flow components.

Particle Image Velocimetry (PIV) Analysis

PIV was initiallydeveloped in the1980s to compute flow fields instantly (Adrian, 1991) and is based on design matching of two successive images irrespective of whether the pictures are obtained from a fluid or solid element (Baba & Peth, 2012). This method employs an optical evaluation system that utilises visible flow and digital images. One benefit of PIV is that it provides instantaneous high-resolution flow velocity vector data of the overall plane in the flow (Stamhuis, 2006).

The main proposal of the PIV method is to obtain consecutive digital images from the sensors employed in digital cameras and video cameras to record standing and moving images. These images can then be analysed by a computer program to determine velocities of the tracer particles. This technique has considerable potential for observing full flow.
It is proposed in the present studyto employ the high potential of the PIV method to formulate the failure mechanism in slopes and compare the results of the proposed model with those of other techniques. A typical case is micro failure modelling, which is required forthe study of micro movements in natural or human-made slopes. These types of failures are dominant in dry, loose soils such as sands with fine particles. The PIV method is utilised to reveal further information regarding shallow failures and their features as a prevalent type of slope stability issue.

III. EXPERIMENTAL METHOD

Aschematic diagram of the experimental method is depicted in Figure 4, which shows the stepwise process utilised to formulate the soil slope stability based on the effective parameters. All stages of the model implementation are systematically described and the results are clearly presented in the following sections.

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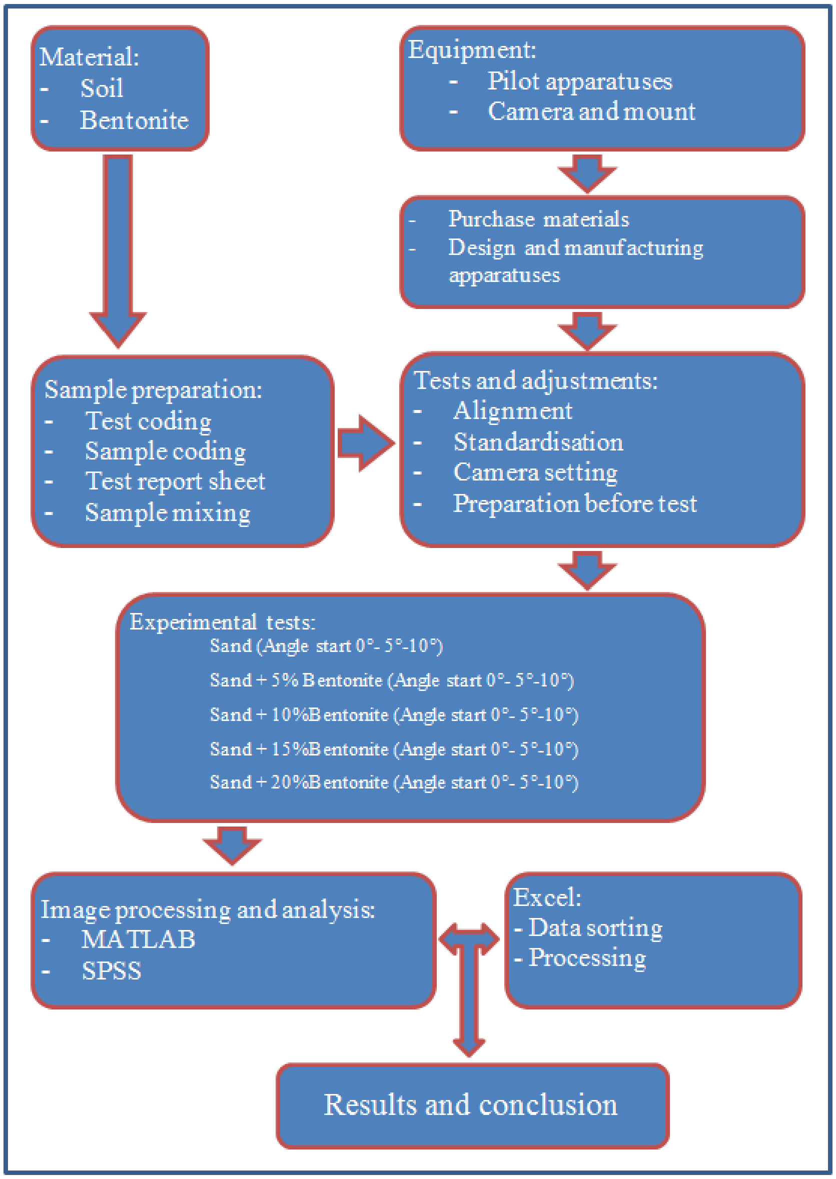

Figure 4: Schematic diagram of the experimental method

1. **Material**

Soil

Soil samples were collected from an area of Gaskell Avenue, Ellenbrook, Western Australia (Figure 5), which belongs to Rocla Quarry Products Pty Ltd.

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Figure 5: Location of the soil resource (GoogleMaps, 2016)

The soil was labelled "Gaskell concrete sand" in the laboratory. To obtain a representative well-mixed sample, a standard collection (AS 1141.3.1-2012) procedure was followed, which involved establishment of a platform where the collected soil was placed and then the soil was mixed thoroughly several times. The soil pile was then divided into four parts to select one part as a representative sample and the remaining three parts were discarded. This procedure was then repeated to obtain the required quantity of soil. Next, the selected soilsample was thoroughly mixed. The discarded parts were placed into bags for transportation to the environmental engineering laboratoryatCurtin University.

Bentonite

In this experiment, bentonite, obtained from Silbelco Australia, was mixed with the soil as time progressed. Typical percentages of bentonite in the soil varied between 5 and 20%.

2. Laboratory Tests

Particle Density

The unit weight of the soil was obtained according to standard AS 1141.5-2000, which is the method for determining particle density, apparent particle density and water absorption of fine aggregates or the fine fraction of an aggregate. The density of sand particles in the present experiment was approximately 2.62 ton/M^3

Direct Shear Test

The direct shear test wasperformed according to standard AS 1289.6.2.2-1998, which is the standard approach for determining the shear strength of a soil (in terms of effective stress) by direct shearing in a shear box.

After extraction, the Mohr Circle calculations are shown inFigure 3 3 (samples SB5-D). Because following Mohr circle graphs exported from edited macro excel file half circle is shown in Figure 6.

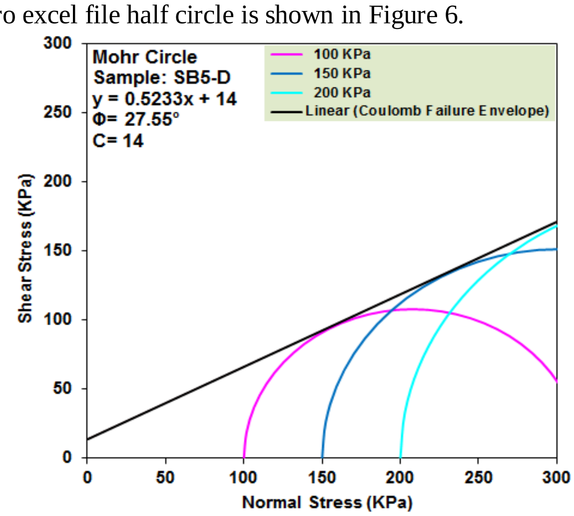

Figure 6: Mohr Circle result for sample SB5-D (5% bentonite)

3. PIV Apparatus Design and Manufacturing

An apparatus design is a small-scale industrial system that helps researchers formulate the behavioural patterns of a system or sub-system for use in the design of a full-scale facility. The apparatus design can be built in different sizes, although it is a relative term in the case that the apparatus is naturally smaller than the full-scale facility. The apparatus design is usually a system established in the laboratory using a stock of laboratory materials. Researchers interchangeably utilise the terms "apparatus design" and "pilot plant"; however, an apparatus design is usually smaller thana pilot plant.

a. An apparatus design can be comprised of the following components:
b. A main framework support structure onto which parts are assembled.
c. A tank container with a clear wall to observemovement of particles inside the tank.
d. A base plate under the tank that lifts from one side and is connected with a hinge on the other side.
e. Several specialised parts to prevent the tank from sliding on the table when one side is lifted.
f. A force generator like a winch or jack to lift one side of the table.
g. A regulator under the legs for alignment purposes.

4. Sample Preparation

As mentioned in the Material section, the main sample considered for testing was sand extracted from Gaskell Avenue, contains the greatestarea ofthis sandin Western Australia; the sand is smooth and, because of considerable impurity effects, bentonite was added. A study by Ata et al. (2015) also utilised a mixture of soil and bentonite because of coherence effect (more details are presented in section 3.2.2 Direct Shear Test). Hashemi et al. (2015) used 10, 15 and 20% bentonite in their testsand Wong et al. (2013) studied further theeffectiveness of bentonite in incoherence. For a drained test, the soil sample should be kept in an oven at a temperature of 105°C for 24 h.

5. Apparatus Standardisation Test

Figure 7presents a schematic diagram of the process of the apparatus standardisation test. One major factor when attempting to solve a problem using image processing is to conduct the test in the dark to prevent light reflection

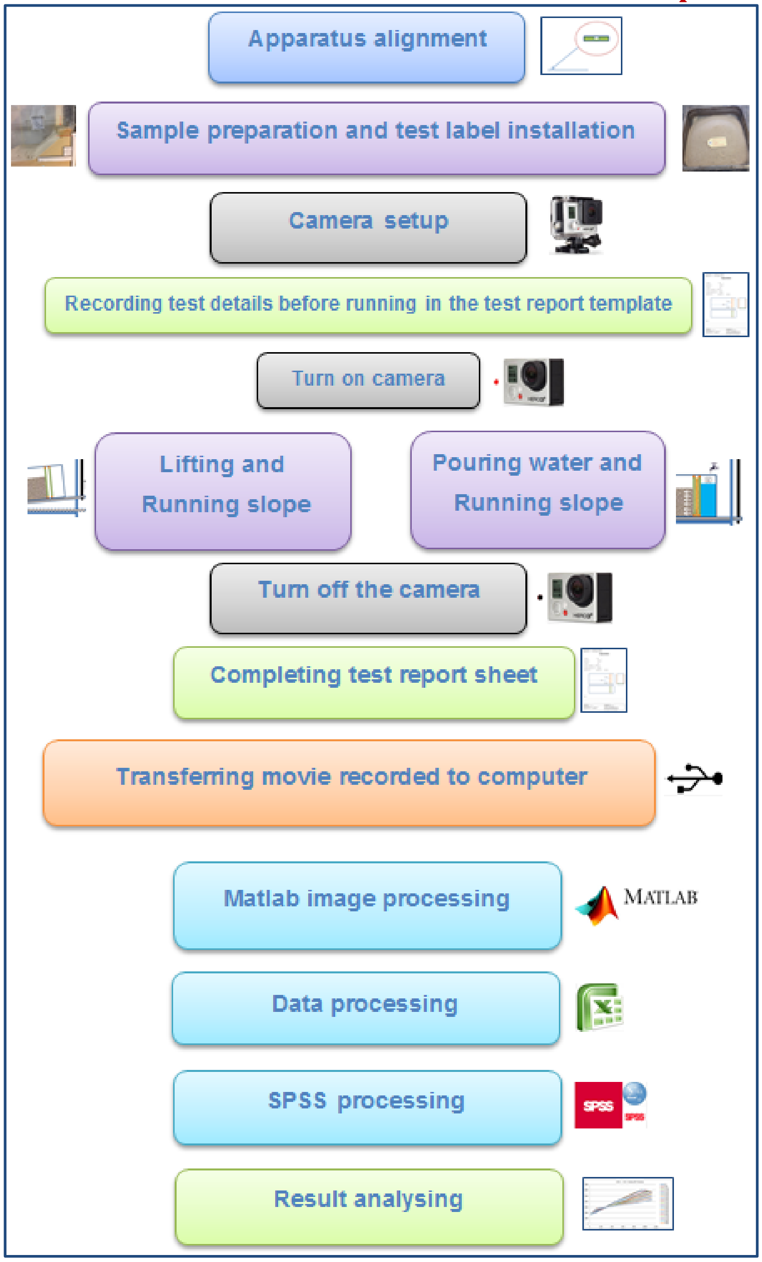

Figure 7: Schematic diagram for apparatus standardisation test

6. Apparatus Alignment

Themain framework and tank were initially aligned in an attempt to begin the tests from different initial slopes to replicate the natural effect of sub-bottom hard layers. Thus, for each test three aligned angles (0°, 5° and 10°) were applied.

7. Statistical Analysis

A comprehensive software package employed for statistical analysis is SPSS. Using the numerical data extracted from the MATLAB software, a comprehensive analysis was accomplished using SPSS software. This software is capable of classifying data with the same characteristics into similar categories. In this research, K-Mean cluster analysis, one of the best classification methods, was applied(IBM, 2013) to analysethe frame category and determine the slope movement jumping in specific frame intervals.

To compare soil slope stability assuming the same ingredients, the displacement value of the soil was computed using the PIV procedure in the MATLAB software program

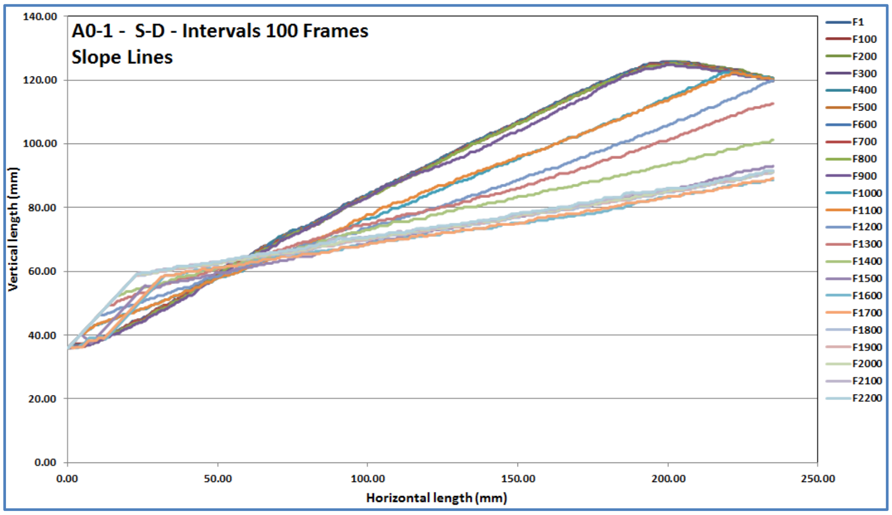

Figure 8: Changes in slope stability for different frames with alignment of 0°

Figure 8depicts different curves of slope stability when the frame numbers change along the test time with an interval of 100 and shows that the slope angle steadily decreased with a gradual increase in the frame numbers or rotation of the tank. This figure illustrates how slope stability can change when the frame numbers are varied or the tank is moved upward at a constant rate of 5 mm/s vertically or 0.95 deg/s rotationally. A movement resistance in the initial frame can be seen followed by a jump in rotations, which can be called the failure threshold.

8. Summary of Results from Image Processing

Bentonite was added to the samples to replicate fine content effects foranalysing slope stability. From the results obtained, by adding bentonite the slope stability decreased significantly. For example, at 5° alignment, the stability of soil with 5% bentonite, soil with 10% bentonite and soil with 15% bentonite steadily decreased. The decrease in the slope inclination can be directly attributed to the increase in volume of bentonite. As presented in the figures, the slope stability curves shifteddownwards with an increase in bentonite content. Because soil displacement varies with composition, an important design problem in slope stability analysis is soil decomposition.

From these results, it can be concluded that soil with no bentonite is more stable than soil with bentonite because as the bentonite content increases the stability of the slope decreases. Whereas, the least stable slope belongedto the sample with the soil and the least alignment. For all curves resulting from the image processing technique, the slope stability decreased with frame numbers. Therefore, the potential for slope stability decreasedwhen the frame number increased. The results indicate that slope stability is a function of the frame number in addition to the degree of composition.
The alignment varied from 0° to 15°, with thisvariable being slightly less significant for changes in the slopes, with the effect of alignment on slope stability being negligible.

9. Conclusion

In the present study, the following research findingswere obtained: (i) the effect of bentonite percentage on slope stability, (ii) development of a graphical model for slope stability analysis, (iii) influence of the alignment on surface failure, and (iv) failure features. The main results are summarised below:

Influence of Bentonite

In this research, the influence of bentonite on the soil slope stability was evaluated, with the bentonite content ranging from 0 to 20%. For different alignments, the slope angle was investigated. The bentonite contents steadily increased for all slope angles and the results are graphically shown. The results revealed that bentonite or dirtiness is not helpful for sandy slopes when they are dry and loose, i.e. planar failures are more unstable with increasing fine particle content in sand.

Benefits of the PIV Method

Fromevaluation of the surface lines extracted from images, it is clear that the mode of failure is a planar failure in a rotational manner. This does not comply with transitional failure mechanisms described in different literature.
Based on the results obtained from the laboratory experiments, the following results are highlighted:

- PIV can be efficiently employed to analyse slope stability in 2D analyses.
- PIV delivers real time changes in slope stability analyses in terms of velocities, safety factors and distribution of failure surfaces.
- When modeling 2D soil slopes, PIV provides a more suitable failure mechanism in comparison tothat of conventional techniques.
- Depending on the complexity of the soil composition that can influence slope stability, conventional techniques are not capable of modelling the complexity involved in the process of movements.
- Based on the obtained results, the significant movement velocity of the slope occurs with reduction of safety factors with a delay of a few seconds after failure.
- From the research analysis, it can be inferred that increases in bentonite content negatively impacts dry slopes. Therefore, considerable increase in the percentage of bentonite has a significant decrease in the slope stability.
- 2D analysis utilisingthe PIV methodis superior to conventional methods because of its versatility and capabilities.

Planar Failure Features

It wasdiscovered that shallow failures in loose sands or clayey sands do not have transitional or planar failure patterns. They have more rotational trends around an axis located on the slope face between toes to one third of its length. It should be noted that this type of slope resists against loading to some extent and shows high velocity of movements approximately 2–4 deg/s when they are approaching failure.

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