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A NOTE ON FG- INTERIOR
J.K. Maitra\textsuperscript{1}, Sujeeet Chaturvedi\textsuperscript{1} & Lakshmi Narayan Mishra\textsuperscript{2}
\textsuperscript{1}Department of Mathematics and Computer Science, R.D. University, Jabalpur 482001
\textsuperscript{2}Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology (VIT) University, Vellore 632 014, Tamil Nadu, India

Corresponding Author\textsuperscript{*} - Lakshmi Narayan Mishra

ABSTRACT
In this paper we have obtained significant properties of Fuzzy g-Interior of a set in fuzzy generalized topological space.

Keywords: Fuzzy sets, Fuzzy topology, Generalized fuzzy g-Interior.

I. INTRODUCTION
The concept of generalized topological spaces was introduced and investigated by A. Csaszar. We introduce a new class of Fuzzy g-Interior of a set in fuzzy generalized topological space. Also we investigate some of their basic properties and produced many interesting theorems.

II. PRELIMINARIES

Definition 2.1 Let X be a (non-empty) universal crisp set. A fuzzy topology on X is a non empty collection \( \tau \) of fuzzy sets on X satisfies the following conditions
(i) Fuzzy sets 0 and 1 belong to \( \tau \)
(ii) Any arbitrary union of members of \( \tau \) is in \( \tau \)
(iii) A finite intersection of members of \( \tau \) is in \( \tau \)

Here 0 and 1 represent the Zero Fuzzy Set and the Whole Fuzzy set on X, defined as, \( 0(x) = 0, \forall x \in X \) and \( 1(x) = 1, \forall x \in X \). The pair \((X, \tau)\) is called Fuzzy Topological Space on X. For Convenience, we shall denote the fuzzy topological space simply as X.

Example 2.1 Let \( X = \{x_1, x_2, x_3\} \) be the universal crisp set and \( \lambda \) be a fuzzy set defined on X as \( \lambda(x_1) = 0.8, \lambda(x_2) = 0.5, \lambda(x_3) = 0.2 \). Then we can see that the collection \( \{0, A, 1\} \) satisfies all the three conditions of fuzzy topology on X. Hence \( \tau = \{0, \lambda, 1\} \) is a fuzzy topology on X and \( \{X, \tau\} \) is a fuzzy topological space.

Proposition 2.1 Let \((X, \tau)\) be a fuzzy topological space and \( \lambda \) let be a fuzzy set in X. Then
(i) \( \phi, x \) are fuzzy closed set in X.
(ii) Arbitrary intersection of fuzzy closed sets is a fuzzy closed set.
(iii) Finite union of fuzzy closed sets is a fuzzy closed set.
proof: (i) let \( \phi \) and \( X \) are fuzzy g-closed set it follow that their complement \( X \) and \( \phi \) are fuzzy g-closed set in \( X \).

Proof: (ii). Let \( (X, \tau) \) be a fuzzy topological space and let \( \left\{ \lambda_j^c \right\}_{j \in J} \) be the collection fuzzy closed sets in \( X \). Where \( J \) is any index set then \( \left\{ \lambda_j^c \right\}_{j \in J} \) is a collection of fuzzy open sets in \( X \). This implies \( \left( \bigcap_{j \in J} \lambda_j^c \right)^c = \left( \bigcap_{j \in J} \lambda_j^c \right)^c \) is a fuzzy closed set in \( X \).

(iii). Let \( \lambda_1, \lambda_2 \) be two fuzzy closed sets in \( X \) this means \( \lambda_1^c \) and \( \lambda_2^c \) are fuzzy open sets in \( X \). Therefore \( \lambda_1^c \lambda_2^c = \left( \lambda_1 \lambda_2 \right)^c \) is a fuzzy open set in \( X \). Hence \( \lambda_1 \lambda_2 \) is a closed set in \( X \).

Definition 2.2 Let \( (X, \tau) \) be a fuzzy topological space and let \( \lambda \) be a fuzzy set in \( X \). Then closure of fuzzy set \( \lambda \) is denoted by \( cl(\lambda) \) and is defined to the intersection of all fuzzy closed sets in \( X \) containing \( \lambda \).

Remark 2.2: We note that \( cl(\lambda) = \inf \{K: \lambda \subseteq K, K^c \in \tau\} \) Thus closure of a fuzzy set \( \lambda \) is the smallest fuzzy closed set containing.

Proposition 2.2 Let \( \lambda \) be a fuzzy set in a fuzzy topological space \( (X, \tau) \), then \( \lambda \) is a fuzzy closed if \( Cl(\lambda) = \lambda \)

Proof: Suppose that \( X \) is a fuzzy closed set in \( X \). Since \( Cl(\lambda) \) is the intersection of all fuzzy closed sets in \( X \) containing \( \lambda \)

And \( \lambda \subseteq \lambda \) follows that \( Cl(\lambda) \subseteq \lambda \). As we know that \( \lambda \subseteq Cl(\lambda) \). Thus, we find that \( Cl(\lambda) = \lambda \).

Conversely, suppose that \( Cl(\lambda) = \lambda \). Then by the definition of closure of fuzzy sets it follows that \( Cl(\lambda) \subseteq \lambda \) is a fuzzy closed set. Thus \( \lambda \) is a fuzzy closed set in \( X \).

Proposition 2.3 Let \( (X, \tau) \) be a fuzzy topological space and \( \lambda_1, \lambda_2 \) be a two fuzzy sets in \( X \). Then:

i Cl(\( \phi \)) = \( \phi \)

ii Cl(\( x \)) = \( x \)

iii if \( \lambda_1, \lambda_2 \) then \( cl(\lambda_1) \subseteq cl(\lambda_2) \)

iv \( cl(\lambda_1 \lambda_2) = cl(\lambda_1) cl(\lambda_2) \)

v \( cl(\lambda_1 \lambda_2) \subseteq cl(\lambda_1) cl(\lambda_2) \)

vi \( cl(cl(\lambda_1)) = cl(\lambda_1) \)
Proposition 2.4: Let $X$ be a topological space and $\{\lambda_j\}_{j \in J}$ be a family of subsets of $X$. Then

(i) $\bigcup_{j \in J} \text{Cl}(\lambda_j) \subseteq \text{Cl}(\lambda_{\bigcup J} \lambda_j)$
(ii) $\text{Cl}(\bigcup_{j \in J} A_j) \subseteq \bigcup_{j \in J} \text{Cl}(\lambda_j)$

Definition 2.3: Let $(X, \tau)$ be a topological space and let $\lambda$ be a fuzzy set in $X$. Then interior of fuzzy set $\lambda$ is denoted by $\text{Int}(\lambda)$ and is defined to be the union of all fuzzy open sets in $X$ which are contained in $\lambda$.

Remark 2.2: We note that $\text{Int}(\lambda) = \text{Sup} \{0:0 \leq \lambda, 0 \in \tau\}$. Thus interior of a fuzzy set $\lambda$ is the largest fuzzy open set contained in $\lambda$.

Proposition 2.5: Let $(X, \tau)$ be a fuzzy topological space and $\lambda_1, \lambda_2$ be two fuzzy sets in $X$. Then:

i. $\text{Int}(\phi) = \phi$

ii. $\text{Int}(x) = x$

iii. if $\lambda_1, \lambda_2$ then $\text{Int}(\lambda_1) \subseteq \text{Int}(\lambda_2)$

iv. $\text{Int}(\lambda_1 \lambda_2) = \text{Int}(\lambda_1) \cap \text{Int}(\lambda_2)$

v. $\text{Int}(\lambda_1) \cap \lambda_2 = \text{Int}(\lambda_1 \lambda_2)$

vi. $\text{Int}(\text{Int}(\lambda_1)) = \text{Int}(\lambda_1)$

Proposition 2.6: Let $(X, \tau)$ be a fuzzy topological space and let $\{\lambda_j\}_{j \in J}$ be the collection fuzzy closed sets in $X$. Where $J$ is any index set then

(i) $\bigcup_{j \in J} \text{int}(\lambda_j) \subseteq \text{int}(\bigcup_{j \in J} \lambda_j)$

(ii) $\text{Int}(\bigcup_{j \in J} \lambda_j) \subseteq \bigcup_{j \in J} \text{Int}(\lambda_j)$

Proposition 2.7: Let $(X, \tau)$ be a fuzzy topological space and $\lambda$ let be a fuzzy set in $X$. Then

(i) $\text{Int}(1 - \lambda) = 1 - \text{Cl}(\lambda)$

(ii) $\text{Cl}(1 - \lambda) = 1 - \text{Int}(\lambda)$

Proof: We have $\text{Int}(\lambda) = \bigcup_{j \in J} \lambda_j$ where $\lambda$ is a fuzzy open set in $X$ and $\lambda_j \leq \lambda, \forall \lambda_j \in J$. This implies that $1 - \text{int}(\lambda) = 1 - \bigcup_{j \in J} \lambda_j = \bigcup_{j \in J} \lambda_j^c$, where $\{\lambda_j^c\}$ is the family of fuzzy closed sets containing $(1 - \lambda)$. Further, we have $\text{Int}(1 - \lambda) = 1 - \text{Cl}(\lambda)$. Hence, by the definition of closure of fuzzy set we get $\text{Cl}(1 - \lambda) = 1 - \text{Int}(\lambda)$.

G-interior in generalized fuzzy topological spaces
Definition 3.1: Let \( X \) be a (non-empty) universal set. A fuzzy topology on \( X \) is a non empty collection \( \tau_{Fg} \) of fuzzy sets on \( X \) satisfying the conditions

(i) Fuzzy sets \( 0 \) and \( 1 \) belong to \( \tau \)

(ii) if \( \{ \lambda_j \} \) for \( j \in J \) is any family of fuzzy sets on \( X \) and \( \lambda_j \in \tau_{Fg}, \forall j \in J \) then \( j \in J \lambda_j \in \tau_{Fg} \)

the pair \( (x, \tau_{Fg}) \) is called fuzzy generalized topological. the element of family \( \tau_{Fg} \) are called fuzzy \( g \)-open sets and their complements are called fuzzy \( g \)-closed sets.

Definitions 3.2

Let \( (X, \tau_{Fg}) \) be a topological space and let \( \lambda \) be a generalized fuzzy set in \( X \). Then the \( g \)-interior of fuzzy set \( \lambda \) is denoted by \( I_{Fg}(\lambda) \) and is defined to be the union of all fuzzy \( g \)-open sets in \( X \) which are contained in \( \lambda \).

Remark 3.2 We note that \( I_{Fg}(\lambda) = \text{Sup} \{0:0 \leq \lambda_0, 0 \in \tau\} \). Thus \( g \)-interior of a fuzzy set \( \lambda \) is the largest fuzzy \( g \)-open set contained in \( \lambda \).

Proposition 3.1 Let \( \lambda \) be a fuzzy set in fuzzy generalized topological space \( (X, \tau_{Fg}) \) then \( \lambda \) is fuzzy \( g \)-open if and only if \( I_{Fg}(\lambda) = \lambda \)

Proof: Suppose \( \lambda \) is a fuzzy \( g \)-open set in \( X \). Since \( I_{Fg}(\lambda) \) is the union of all fuzzy \( g \)-open sets in \( X \) contained in \( \lambda \) and \( \lambda \leq \lambda \) follows that \( I_{Fg}(\lambda) \leq \lambda \) As we know that \( I_{Fg}(\lambda) \leq \lambda \). Thus we find that \( I_{Fg}(\lambda) = \lambda \). Conversely, suppose that \( I_{Fg}(\lambda) \) then by definition of \( g \)-interior of fuzzy set, it follows that \( I_{Fg}(\lambda) \) is a fuzzy \( g \)-open set. Thus \( \lambda \) is a fuzzy \( g \)-open set in \( X \).

Proposition 3.2 Let \( (X, \tau_{Fg}) \) be a generalized fuzzy topological space and \( \lambda_1, \lambda_2 \) be two fuzzy sets in \( X \). Then ;

i \( I_{Fg}(\phi) = \phi \)

ii \( I_{Fg}(x) = x \)

iii if \( \lambda_1, \lambda_2 \) then \( I_{Fg}(\lambda) \subseteq I_{Fg}(\lambda) \)

iv \( I_{Fg}(\lambda_1 \cap \lambda_2) = I_{Fg}(\lambda_1) \cap I_{Fg}(\lambda_2) \)

v \( I_{Fg}(\lambda_1) \cap I_{Fg}(\lambda_2) \subseteq I_{Fg}(\lambda_2 \cap \lambda_2) \)

vi \( I_{Fg}(I_{Fg}(\lambda_1)) = I_{Fg}(\lambda_1) \)
proof: (i) Since $\phi$ and $X$ are fuzzy g-open sets from let $(x, \mathcal{T}_{Fg})$ be a generalized fuzzy topological space and let $\lambda$ be a fuzzy set in $X$. Then $\lambda$ is fuzzy g-open set if and only if $I_{Fg}(\lambda) = \lambda$. We have $I_{Fg}(\phi) = \phi$ and $I_{Fg}(X) = X$.

(ii) Suppose $\lambda_1 \subseteq \lambda_2$ in $X$ since $\lambda_2 \subseteq I_{Fg}(\lambda_2)$ and $\lambda_1 \subseteq \lambda_2$ we have $\lambda_1 \subseteq I_{Fg}(\lambda_2)$. Now $I_{Fg}(\lambda_2)$ is a fuzzy g-open set and is $I_{Fg}(\lambda_1)$ the largest fuzzy g-open set containing $\lambda$. We find that $I_{Fg}(\lambda_1) \subseteq I_{Fg}(\lambda_2)$.

(iii) Since $\lambda_1 \subseteq \lambda_2$, $\lambda_2 \subseteq \lambda_1$ we have $I_{Fg}(\lambda_1) \subseteq I_{Fg}(\lambda_1, \lambda_2)$ and $I_{Fg}(\lambda_2) \subseteq I_{Fg}(\lambda_1, \lambda_2)$. This implies $I_{Fg}(\lambda_1) \subseteq I_{Fg}(\lambda_1, \lambda_2)$.

(iv) Since $\lambda_1, \lambda_2 \subseteq \lambda_1$ and $\lambda_1, \lambda_2 \subseteq \lambda_2$ we have $I_{Fg}(\lambda_1, \lambda_2) \subseteq I_{Fg}(\lambda_1) \cap I_{Fg}(\lambda_2)$.

(v) Since $I_{Fg}(\lambda_i)$ is a g-fuzzy open set in $X$, if follow that $I_{Fg}(I_{Fg}(\lambda_1)) = I_{Fg}(\lambda_1)$.

Example 3.1 Let $X = \{x_1, x_2\}$ let $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ be fuzzy sets for $X$ defined as:

- $\lambda_1 = \{(x_1, 0.4), (x_2, 0.6)\}$
- $\lambda_2 = \{(x_1, 0.5), (x_2, 0.3)\}$
- $\lambda_3 = \{(x_1, 0.3), (x_2, 0.4)\}$
- $\lambda_4 = \{(x_1, 0.5), (x_2, 0.6)\}$

Then $\mathcal{T}_{Fg} = \{0, \lambda_1, \lambda_2, \lambda_3, \lambda_4, 1\}$ is a fuzzy topology on $X$. It is easy to see that $\mathcal{T}_{Fg}$ the collection of closed sets in $X$ is given by:

- $I_{Fg}(\lambda_1^c) = (x_1, 0.5), (x_2, 0.3) = \lambda_2$
- $I_{Fg}(\lambda_2^c) = (x_1, 0.5), (x_2, 0.6) = \lambda_3$
- $I_{Fg}(\lambda_3) = \{(x_1, 0.4), (x_2, 0.6)\} = \lambda_1$
- $I(\lambda_2) = \{(x_1, 0.5), (x_2, 0.3)\} = \lambda_3$

and

- $I(\lambda_1, \lambda_2) = \lambda_3$
proposition 3.3 Let \( (X, \tau_{Fg}) \) be a generalized fuzzy topological space and let \( \{\lambda_j\}_{j \in J} \) be the collection of fuzzy g-open sets in \( X \). Where \( J \) is any index set then

\[(i) \quad I_{Fg} \left( \bigcup_{j \in J} \lambda_j \right) \subseteq I_{Fg} (\lambda) \]

\[(ii) \quad I_{Fg} \left( \bigcap_{j \in J} \lambda_j \right) \subseteq \bigcap_{j \in J} I_{Fg} (\lambda) \]

**Proposition 3.4** Let \( (X, \tau_{Fg}) \) be a generalized fuzzy topological space and \( \lambda \) let be a fuzzy set in \( X \). Then

\[(i) \quad I_{Fg} (1 - \lambda) = 1 - C_{Fg} (\lambda) \]

\[(ii) \quad C_{Fg} (1 - \lambda) = 1 - I_{Fg} (\lambda) \]

**Proof:** We have \( C_{Fg} (1 - \lambda) = 1 - I_{Fg} (\lambda) \) where \( \lambda \) is a fuzzy open set in \( X \) and \( \lambda_j \leq \lambda, \forall j \in J \).

This implies that \( 1 - I_{Fg} (\lambda) = 1 - \bigcup_{j \in J} \lambda_j = \lambda^{\text{c}}, \) where \( \{\lambda^c\} \) is the family of fuzzy g-open sets containing \( (1 - \lambda) \). Further, we have this implies \( I_{Fg} (1 - \lambda) = C_{Fg} (1 - \lambda) \). Hence, by the definition of g-open of fuzzy set we get \( C_{Fg} (1 - \lambda) = 1 - I_{Fg} (\lambda) \).

**REFERENCES**