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NANO CONTRA αψ CONTINUOUS AND NANO CONTRA αψ IRRESOLUTE IN NANO TOPOLOGY

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ABSTRACT
The intention of this learning is to introduce the idea of nano contra αψ continuous and nano contra αψ irresolute functions and examine some of their associated attributes and theorems. The corresponding condition for a function to be nano contra αψ continuous and nano contra αψ irresolute functions is established.

Keywords: Nano αψ kernal, Nano contra αψ continuous, Nano contra αψ irresolute.
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I. INTRODUCTION
Generalized closed sets in topology was introduced by Levine [13]. Semi generalized closed sets in topology was introduced by Bhattacharyya and Lahiri [1] and study of their related attributes. A new concept of nano topology was introduced by Lellis Thivagar [10,11]. He also investigated some of their properties like nano open, nano semi open and nano pre open sets in nano topological spaces. K.Bhuvaneswari [2,3] etal was introduced the new concept of nano generalized closed and nano semi generalized closed sets in nano topology. A new class of contra continuity in nano topology was introduced by Lellis Thivagar [12]. S.Chandrasekar [5] etal present a new concept of contra nano semi generalized continuous in nano topology.

In this article, we will introduce the concept of nano contra αψ continuous and nano contra αψ irresolute functions and investigate some of their related attributes and theorems.

II. PRELIMINARIES

Definition 2.1. [10] Let U be a non empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Let X is a subset of U, then the lower approximation of X with respect to R is is denoted by

\[ R = \bigcup_{x \in U} \{ R(x) : R(x) \subseteq X \} \]

where \( R(x) \) denotes the equivalence class determined by \( x \in U \).

Definition 2.2. [10] The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and its is denoted by

\[ \overline{R} = \bigcup_{x \in U} \{ R(x) : R(x) \cap X \neq \emptyset \} \]

Definition 2.3. [10] The boundary region of X with respect to R is the set of all objects, which can be possibly classified neither as X nor as not X with respect to R and its is denoted by \( B_R = \overline{R} - R \).
Definition 2.4. [10] If \((U, R)\) is an approximation space and \(X, Y \subseteq U\). Then

1. \(R \subseteq X \subseteq \overline{R}\)
2. \(\overline{R(\phi)} = \overline{R(\phi)} = \phi\) and \(\overline{R(U)} = \overline{R(U)} = U\)
3. \(\overline{R(X \cup Y)} = \overline{R(X \cup Y)}\)
4. \(\overline{R(X \cap Y)} = \overline{R(X \cap Y)}\)
5. \(\overline{R(X \cup Y)} = \overline{R(X \cup Y)}\)
6. \(\overline{R(X \cap Y)} = \overline{R(X \cap Y)}\)
7. \(\overline{R(X)} \subseteq \overline{R(Y)}\) and \(\overline{R(X)} \subseteq \overline{R(Y)}\) whenever \(X \subseteq Y\)
8. \(\overline{R(X^c)} = \overline{R(X^c)}\) and \(\overline{R(X^c)} = \overline{R(X^c)}\)
9. \(\overline{R(R)} = \overline{R(R)} = R\)
10. \(\overline{\overline{R(R)}} = \overline{R(R)} = \overline{R}\)

Definition 2.5. [10] Let \(U\) be an universe and \(R\) be an equivalence relation on \(U\) and \(\tau_R(X) = \{U, \phi, \overline{R}, \overline{R}, B_R\}\) where \(X \subseteq U\). Then \(\tau_R(X)\) satisfies the following axioms:

1. \(U\) and \(\phi \in \tau_R(X)\).
2. The union of the element of any sub collection of \(\tau_R(X)\) is in \(\tau_R(X)\).
3. The intersection of the element of any finite sub collection of \(\tau_R(X)\) is in \(\tau_R(X)\).

Then \(\tau_R(X)\) is a topology on \(U\) is called the nano topology on \(U\) with respect to \(X\). \((U, \tau_R(X))\) as the nano topological space. The element of \(\tau_R(X)\) are called as nano open sets and complement of nano open sets is called nano closed.

III. NANO CONTRA \(\alpha\psi\) CONTINUOUS FUNCTION

Definition 3.1. Suppose \((s, \tau_R(x))\) be a nano topological spaces and \(H \subseteq S\). Then the nano \(\alpha\psi\) kernel of \(H\) is defined by \(N_{\alpha\psi}\ker(H) = \bigcap \{S : H \subseteq S, S \in \tau_R(\alpha\psi)(x)\}\).

Example 3.2. Let us consider \(U = \{p, q, r, s\}\) with \(U/R = \{\{p\}, \{q\}, \{r\}, \{s\}\}\) and \(X = \{p, q\}\). Then \(\tau_R(X) = \{U, \phi, \{p\}, \{p, q, s\}, \{q, s\}\}\). Here the nano \(\alpha\psi\) open set is \(\tau_R(\alpha\psi)(X) = \{U, \phi, \{p\}, \{q\}, \{r\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}, \{q, r, s\}\}\). Hence \(N_{\alpha\psi}\ker\{p\} = \{p\}, N_{\alpha\psi}\ker\{p, q, r\} = \{p, q, r\}\) and \(N_{\alpha\psi}\ker\{r\} = \{U\}\).

Theorem 3.3. Suppose \((s, \tau_R(x))\) be a nano topological spaces and \(H, R \subseteq S\). Then the subsequent attributes hold.

(i) \(a \in N_{\alpha\psi}\ker(H)\) if and only if \(H \cap E \neq \phi\) for any nano \(\alpha\psi\) closed containing \(a\).

(ii) If \(H\) is a subset of \(N_{\alpha\psi}\ker(H)\) and then \(H = N_{\alpha\psi}\ker(H)\) if \(H\) is nano \(\alpha\psi\) open.

(iii) If \(H\) is a subset of \(R\), then \(N_{\alpha\psi}\ker(H)\) is a subset of \(N_{\alpha\psi}\ker(R)\).
(i) \( \Rightarrow \) If \( a \in N_{\alpha\psi} \ker(H) \). To prove that \( H \cap E \neq \phi \) for any nano \( \alpha\psi \) closed containing \( a \). Let \( a \in N_{\alpha\psi} \ker(H) \), then \( x \in H \subseteq E^c \), where \( E^c \) is a nano \( \alpha\psi \) open set and \( \Rightarrow H \cap E^c \neq \phi \). Hence \( H \subseteq E^c \subseteq E \Rightarrow H \cap E \neq \phi \), here \( E \) is nano \( \alpha\psi \) closed. Therefore \( a \in H \cap E \neq \phi \), \( E \) is nano \( \alpha\psi \) closed. Hence \( H \cap E \neq \phi \) for any nano \( \alpha\psi \) closed set containing \( a \).

Conversely, If \( H \cap E \neq \phi \) for any nano \( \alpha\psi \) closed set containing \( a \). To prove that, \( a \in N_{\alpha\psi} \ker(H) \).

Assume that \( a \notin N_{\alpha\psi} \ker(H) \), hence there exist a nano \( \alpha\psi \) open set \( E^c \) such that \( H \subseteq E^c \) and \( a \notin E^c \). Hence \( H \subseteq E \) and \( a \notin E \), where \( E \) is a nano \( \alpha\psi \) closed which is a contradiction. Therefore \( a \in N_{\alpha\psi} \ker(H) \).

(ii) \( \Rightarrow \) If \( H \) is a nano \( \alpha\psi \) open set then \( N_{\alpha\psi} \ker(H) \subseteq H \). To prove that, \( H = N_{\alpha\psi} \ker(H) \).

Let us take \( R \) be any nano \( \alpha\psi \) open set containing \( H \), then we have \( H \subseteq K \), implies \( H \subseteq K \cap H \subseteq H \) and \( K \cap H \) is nano \( \alpha\psi \) open set. Therefore \( H \subseteq \bigcap K, H \subseteq K, K \in \tau_{\alpha\psi}^{a,\psi}(x) \). Hence \( H \subseteq N_{\alpha\psi} \ker(H) \) this implies that \( H = N_{\alpha\psi} \ker(H) \).

(iii) \( \Rightarrow \) If \( H \subseteq S \), then to prove that \( N_{\alpha\psi} \ker(H) \subseteq N_{\alpha\psi} \ker(S) \).

Let \( P \in N_{\alpha\psi} \ker(H) \Rightarrow H \subseteq P \) and \( P \) is nano \( \alpha\psi \) open in nano topology. If \( H \subseteq S \) then \( H \subseteq S \subseteq P \) where \( P \) is nano \( \alpha\psi \) open in nano topology. Hence \( P \in N_{\alpha\psi} \ker(S) \). Therefore \( P \in N_{\alpha\psi} \ker(H) \Rightarrow P \in N_{\alpha\psi} \ker(S) \) this implies \( N_{\alpha\psi} \ker(H) \subseteq N_{\alpha\psi} \ker(S) \).

**Theorem 3.4.** Let \( H \) be a subset of nano topology, then the subsequent conditions are equivalent.

(i) \( H \) is nano \( \alpha\psi \) closed in nano topology.

(ii) \( N_{\alpha\psi}(H) \subseteq N_{\alpha\psi}(S) \).

**Proof.**

(i) \( \Rightarrow \) (ii) Suppose \( a \notin N_{\alpha\psi}(H) \), then there exist a set \( S \in N_{\alpha\psi} \) open set in nano topology such that \( a \notin S \) and \( H \subseteq S \). Here \( H \) is nano \( \alpha\psi \) closed, by definition \( N_{\alpha\psi}(H) \subseteq U \) and so \( a \notin N_{\alpha\psi}(H) \). This shows that \( N_{\alpha\psi}(H) \subseteq N_{\alpha\psi}(H) \).

(ii) \( \Rightarrow \) (i) Let \( S \in N_{\alpha\psi} \) open such that \( H \subseteq S \). Then \( N_{\alpha\psi}(H) \subseteq S \) by (ii) \( N_{\alpha\psi}(H) \subseteq S \). Hence \( H \) is nano \( \alpha\psi \) closed by definition.

**Definition 3.5.** Let \( (S, \tau_{\alpha\psi}(x)) \) and \( (T, \tau_{\alpha\psi}(y)) \) be a nano topological spaces, then \( c \) \( f : (S, \tau_{\alpha\psi}(x)) \rightarrow (T, \tau_{\alpha\psi}(y)) \) is a nano contra \( \alpha\psi \) continuous, if \( f^{-1}(P) \) is nano \( \alpha\psi \) closed in \( (S, \tau_{\alpha\psi}(x)) \) for every nano open set \( P \in (T, \tau_{\alpha\psi}(y)) \).

**Example 3.6.** Let as consider \( U = \{p, q, r, s\} \) with \( U/R = \{[p, q], [r], [s]\} \) and \( X = \{p, r\} \). Then \( \tau_{\alpha\psi}(X) = \{U, \phi, \{r\}, \{p, q, r\}, \{p, q\}\} \). Here the nano \( \alpha\psi \) open set is \( \tau_{\alpha\psi}(X) = P(U) \) and \( U = \{p, q, r, s\} \) with \( U/R = \{[p], [r], [q, s]\} \) and \( Y = \{p, q\} \). Then \( \tau_{\alpha\psi}(Y) = \{U, \phi, \{p\}, \{p, q, s\}, \{q, s\}\} \). Here the nano \( \alpha\psi \) open set is \( \tau_{\alpha\psi}(Y) = \{U, \phi, \{p\}, \{q\}, \{p, q\}, \{q, s\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}, \{q, r, s\}\} \). Assume that
Let $P$ be a nano open set in $\mathcal{T}_R(x)$. Assume $f$ is nano contra semi continuous. So $f^{-1}(P)$ is nano semi closed set in $C(\mathcal{T}_R(x))$. We know that each nano semi closed sets is nano $\alpha\psi$ closed set. Hence $f^{-1}(P)$ is nano $\alpha\psi$ closed in $(\mathcal{T}_R(x))$. Therefore $f$ is nano contra $\alpha\psi$ continuous function.

The inverse part need not be true in the following example.

Example 3.8. Let $U = \{p, q, r, s\}$ with $U_R = \{(p, q), \{r\}, \{s\}\}$ and $X = \{p, r\}$. Then $\mathcal{T}_R(x) = \{U, \phi, \{r\}, \{p, q, r\}, \{p, q\}\}$. Here the nano $\alpha\psi$ open set is $\mathcal{T}_R^{\alpha\psi}(X) = P(U)$ and $U = \{p, q, r, s\}$ with $U_R = \{(p, q), \{r\}, \{s\}\}$ and $Y = \{p, q\}$. Then $\mathcal{T}_R(Y) = \{U, \phi, \{p, q, s\}, \{q, s\}\}$. Here the nano $\alpha\psi$ open set is $\mathcal{T}_R^{\alpha\psi}(Y) = \{U, \phi, \{p, q, s\}, \{q, s\}\}$. Assume that $f : (\mathcal{T}_R(x)) \rightarrow (T, \mathcal{T}_R(y))$ be defined by $f(p) = q, f(q) = r, f(r) = s, f(s) = p$. Then $f^{-1}(p, q, r) = \{p, r\}$ is nano contra $\alpha\psi$ continuous in $(s, \mathcal{T}_R(x))$ but it is not nano semi closed in $(s, \mathcal{T}_R(x))$. Therefore $f$ is nano contra $\alpha\psi$ continuous function.

Theorem 3.9. Each nano contra $\alpha$ continuous function is nano contra $\alpha\psi$ continuous function.

Proof. Let $P$ be a nano open set in $(T, \mathcal{T}_R(y))$. Assume $f$ is nano contra $\alpha$ continuous, So $f^{-1}(P)$ is nano $\alpha$ closed set in $C(\mathcal{T}_R(x))$. We know that each nano $\alpha$ closed sets is nano $\alpha\psi$ closed set. Hence $f^{-1}(P)$ is nano $\alpha\psi$ closed in $(\mathcal{T}_R(x))$. Therefore $f$ is nano contra $\alpha\psi$ continuous function.

The inverse part need not be true in the following example.

Example 3.10. Let $U = \{p, q, r, s\}$ with $U_R = \{(p, q), \{r\}, \{s\}\}$ and $X = \{p, r\}$. Then $\mathcal{T}_R(x) = \{U, \phi, \{r\}, \{p, q, r\}, \{p, q\}\}$. Here the nano $\alpha\psi$ open set is $\mathcal{T}_R^{\alpha\psi}(X) = P(U)$ and $U = \{p, q, r, s\}$ with $U_R = \{(p, q), \{r\}, \{s\}\}$ and $Y = \{p, q\}$. Then $\mathcal{T}_R(Y) = \{U, \phi, \{p, q, s\}, \{q, s\}\}$. Here the nano $\alpha\psi$ open set is $\mathcal{T}_R^{\alpha\psi}(Y) = \{U, \phi, \{p, q, s\}, \{q, s\}\}$. Assume that $f : (\mathcal{T}_R(x)) \rightarrow (T, \mathcal{T}_R(y))$ be defined by $f(p) = q, f(q) = r, f(r) = s, f(s) = p$. Then $f^{-1}(p, q, r) = \{p, r\}$ is nano contra $\alpha\psi$ continuous in $(s, \mathcal{T}_R(x))$ but it is not nano $\alpha$ closed in $(s, \mathcal{T}_R(x))$. Therefore $f$ is nano contra $\alpha\psi$ continuous function.

Theorem 3.11. Each nano contra $\psi$ continuous function is nano contra $\alpha\psi$ continuous function.

Proof. Let $P$ be a nano open set in $(T, \mathcal{T}_R(y))$. Assume $f$ is nano contra $\psi$ continuous, So $f^{-1}(P)$ is nano $\psi$ closed set in $(\mathcal{T}_R(x))$. We know that each nano $\psi$ closed sets is nano $\alpha\psi$ closed set. Hence $f^{-1}(P)$ is nano $\alpha\psi$ closed in $(\mathcal{T}_R(x))$. Therefore $f$ is nano contra $\alpha\psi$ continuous function.

The inverse part need not be true in the following example.
Example 3.12. Let us consider $U = \{p,q,r,s\}$ with $U/R = \{[p,q],[r],[s]\}$ and $X = \{p,r\}$. Then

$\tau(X) = \{U,\emptyset,\{p\},\{q\},\{p,q\}\}$. Here the nano $\alpha\psi$ open set is $\tau(X) = P(U)$ and

$U = \{p,q,r,s\}$ with $U/R = \{[p],[r],[q],[s]\}$ and $Y = \{p,q\}$. Then $\tau(Y) = \{U,\emptyset,\{p,q\},\{r\},\{q\}\}$. Here the nano $\alpha\psi$ open set is $\tau(Y) = \{U,\emptyset,\{p,q\},\{r\},\{q\}\}$. Assume that $f : (S, \tau(X)) \to (T, \tau(Y))$ be defined by $f(p) = q, f(q) = r, f(r) = s, f(s) = p$. Then

$f^{-1}(p,q,s) = \{p,r,s\}$ is nano contra $\alpha\psi$ continuous in $(s, \tau(X))$ but it is not nano $\psi$ closed in $(s, \tau(X))$ for every nano open set $\{p,q,s\}$ in $(T, \tau(Y))$.

Theorem 3.13. Each nano contra $\alpha\psi$ continuous function is nano contra $\alpha\psi$ continuous function.

Proof. Let $P$ be a nano open set in $(T, \tau(Y))$. Assume $f$ is nano contra $\alpha\psi$ continuous, So $f^{-1}(P)$ is nano $\alpha\psi$ closed set in $C(S, \tau(X))$. We know that each nano $\alpha\psi$ closed sets is nano $\alpha\psi$ closed set. Hence $f^{-1}(P)$ is nano $\alpha\psi$ closed in $(S, \tau(X))$. Therefore $f$ is nano contra $\alpha\psi$ continuous function.

The inverse part need not be true in the following example.

Example 3.14. Let us consider $U = \{p,q,r,s\}$ with $U/R = \{[p,q],[r],[s]\}$ and $X = \{p,r\}$. Then

$\tau(X) = \{U,\emptyset,\{p\},\{q\},\{p,q\}\}$. Here the nano $\alpha\psi$ open set is $\tau(X) = P(U)$ and

$U = \{p,q,r,s\}$ with $U/R = \{[p],[r],[q],[s]\}$ and $Y = \{p,q\}$. Then $\tau(Y) = \{U,\emptyset,\{p,q\},\{r\},\{q\}\}$. Here the nano $\alpha\psi$ open set is $\tau(Y) = \{U,\emptyset,\{p,q\},\{r\},\{q\}\}$. Assume that $f : (S, \tau(X)) \to (T, \tau(Y))$ be defined by $f(p) = q, f(q) = r, f(r) = s, f(s) = p$. Then

$f^{-1}(p,q,s) = \{p,r,s\}$ is nano contra $\alpha\psi$ continuous in $(S, \tau(X))$ but it is not nano $\alpha\psi$ closed in $(S, \tau(X))$ for every nano open set $\{p,q,s\}$ in $(T, \tau(Y))$.

Theorem 3.15. Let a map $f : (S, \tau(X)) \to (T, \tau(Y))$, then the subsequent attributes are equivalent.

(i) $f$ is nano contra $\alpha\psi$ continuous.

(ii) The inverse image of every nano closed set in $T$ is nano $\alpha\psi$ open in $S$.

(iii) If every nano $\alpha\psi$ open set $H$ in $S$ then $f(H) \subseteq R$, where $R$ is a nano $\alpha\psi$ closed set, $f(a) \in R$ such that every $a \in S$.

(iv) $f(N_{\alpha\psi}(\text{cl}(H))) \subseteq N_{\alpha\psi}(\ker f(H))$ for each subset $H$ of $S$.

(v) $N_{\alpha\psi}(\text{cl}(f^{-1}(R))) \subseteq f^{-1}[N_{\alpha\psi}(\ker f(R))]$.

Proof.

(i) $\Rightarrow$ (ii) Let $f$ be a nano contra $\alpha\psi$ continuous. Assume $R$ be a nano closed set in $T$ and $R^c$ is nano open set in $T$, by $f^{-1}(R^c)$ is a nano $\alpha\psi$ closed set in $S$. But $f^{-1}(R^c) = f^{-1}(R)^c$. Therefore $f^{-1}(B)$ is nano open in $S$.

(ii) $\Rightarrow$ (i) Suppose $R$ be a nano closed set such that $f(a) \in R$, by (ii) $a \in f^{-1}(R)$ which is nano open. Assume $H = f^{-1}(R)$ then $a \in H$ and $f(H) \subseteq R$.

(iii) $\Rightarrow$ (ii) Assume that $R$ be a nano closed set in $T$ and $a \in f^{-1}(R)$ then $f(a) \in R$. there exist a nano open set $S$ such that $f(S) \subseteq R$. Hence $f^{-1}(R)$ is equal to union of all nano $\alpha\psi$ open set.
(iii) \(\Rightarrow\) (iv) Suppose \(H\) be a subset of \(S\). If \(b \notin N_{\alpha\psi}\ker f(H)\), then by Theorem 2.3 there exist \(R \subseteq (T, f(a)) \ni f(H) \cap R = \phi\). Thus \(H \cap f^{-1}(R) = \phi\) and since \(f^{-1}(R)\) is nano open we have \(N_{\alpha\psi}cl(H) \cap f^{-1}(B) = \phi\). Hence, \(f(N_{\alpha\psi}cl(H)) \cap R = \phi\) and therefore \(b \notin f(N_{\alpha\psi}\ker(H)) \Rightarrow f(N_{\alpha\psi}cl(H)) \subseteq N_{\alpha\psi} ker f(H)\).

(iv) \(\Rightarrow\) (v) Let \(R \subseteq T\), by (iv) and Theorem 2.3 we have \(f(N_{\alpha\psi}cl(f^{-1}(R))) \subseteq N_{\alpha\psi} ker f(f^{-1}(R)) \subseteq N_{\alpha\psi} ker(R)\). Thus \(N_{\alpha\psi}cl(f^{-1}(R)) \subseteq f^{-1}[N_{\alpha\psi} ker(R)]\).

(v) \(\Rightarrow\) (i) Let \(R \subseteq T\), then by Theorem 2.3 we have \(N_{\alpha\psi}cl(f^{-1}(R)) \subseteq f^{-1}[N_{\alpha\psi} ker(R)]\) and \(N_{\alpha\psi}cl(f^{-1}(R)) \subseteq f^{-1}(R)\). Therefore \(f^{-1}(R)\) is nano closed in \(S\). Hence \(f\) is nano contra \(\alpha\psi\) continuous.

**Theorem 3.16.** If \(f : S \rightarrow T\) and \(g : T \rightarrow E\) be the function then \(g \circ f\) is nano contra \(\alpha\psi\) continuous if \(g\) is nano \(\alpha\psi\) continuous and \(f\) is nano contra \(\alpha\psi\) continuous.

**Proof.** Let us assume that \(g\) is nano \(\alpha\psi\) continuous and \(f\) is nano contra \(\alpha\psi\) continuous function. Let us take any nano open set \(R\) in \(E\). Here \(g\) is nano \(\alpha\psi\) continuous, therefore \(g^{-1}(R)\) is nano open in \(T\). Here \(f\) is nano contra \(\alpha\psi\) continuous, \(f^{-1}(g^{-1}(R))\) is nano contra \(\alpha\psi\) continuous in \(S\). That is \((g \circ f)^{-1}(R)\) is nano \(\alpha\psi\) continuous in \(S\). Hence \(g \circ f\) is nano contra \(\alpha\psi\) continuous.

**IV. NANO CONTRA \(\alpha\psi\) IRRESOLUTE FUNCTION**

**Definition 4.1.** Let \((S, \tau_{R}(x))\) and \((T, \tau_{R}(y))\) be a nano topological spaces, then \(f : (S, \tau_{R}(x)) \rightarrow (T, \tau_{R}(y))\) is a nano contra \(\alpha\psi\) irresolute function, if \(f^{-1}(P)\) is nano \(\alpha\psi\) closed in \((S, \tau_{R}(x))\) for every nano \(\alpha\psi\) open set \(P\) in \((T, \tau_{R}(y))\).

**Example 4.2.** Let as consider \(U = \{p, q, r, s\}\) with \(U_{/R} = \{[p, q], [r], [s]\}\) and \(X = \{p, r\}\). Then \(\tau_{R}(X) = [U, \phi, \{r\}, \{p, q, r\}, \{p, q\}]\). Here the nano \(\alpha\psi\) open set is \(\tau_{R}^{\alpha\psi}(X) = P(U)\) and \(U = \{p, q, r, s\}\) with \(U_{/R} = \{[p], [q], [q, s]\}\) and \(Y = \{p, q\}\). Then \(\tau_{R}(Y) = [U, \phi, \{p\}, \{p, q, s\}, \{q, s\}]\). Here the nano \(\alpha\psi\) open set is \(\tau_{R}^{\alpha\psi}(Y) = [U, \phi, \{p\}, \{q, s\}, \{q, r\}, \{p, r, s\}, \{q, r, s\}]\). Assume that \(f : (S, \tau_{R}(x)) \rightarrow (T, \tau_{R}(y))\) be defined by \(f(p) = q, f(q) = r, f(r) = s, f(s) = p\). Then \(f^{-1}(p, q, r) = \{p, q, s\}\) is nano \(\alpha\psi\) closed in \((s, \tau_{R}(x))\) for every nano \(\alpha\psi\) open set \(\{p, q, r, s\}\) in \((T, \tau_{R}(y))\).

**Theorem 4.3.** Let \(f\) and \(g\) be a two nano contra \(\alpha\psi\) irresolute function on \(U\) then \(g \circ f\) is need not be a nano contra \(\alpha\psi\) irresolute function.

**Proof.** This is proved by the following example.

**Example 4.4.** Let as consider \(U = \{p, q, r, s\}\) with \(U_{/R} = \{[p, q], [r], [s]\}\) and \(X = \{p, r\}\). Then \(\tau_{R}(X) = [U, \phi, \{r\}, \{p, q, r\}, \{p, q\}]\). Here the nano \(\alpha\psi\) open set is \(\tau_{R}^{\alpha\psi}(X) = P(U)\) and \(U = \{p, q, r, s\}\) with \(U_{/R} = \{[p], [r], [q, s]\}\) and \(Y = \{p, q\}\). Then \(\tau_{R}(Y) = [U, \phi, \{p\}, \{p, q, s\}, \{q, s\}]\). Here the nano \(\alpha\psi\) open set is \(\tau_{R}^{\alpha\psi}(Y) = [U, \phi, \{p\}, \{q, s\}, \{q, r\}, \{p, r, s\}, \{q, r, s\}]\).
Assume that \( f : (S, \tau_R(x)) \rightarrow (T, \tau_R(y)) \) be defined by \( f(p) = q, f(q) = r, f(r) = s, f(s) = p \) and also let \( g : (T, \tau_R(y)) \rightarrow (E, \tau_R(z)) \) be defined by \( g(p) = p, g(q) = r, g(r) = s, g(s) = q \).

Here the functions \( f \) and \( g \) is nano contra \( \alpha \psi \) irresolute functions but their composite \( g \circ f \) is not nano contra \( \alpha \psi \) irresolute function since \( f^{-1}(g^{-1}(p, q, r)) = (p, q, s) \) is not nano \( \alpha \psi \) closed in \( (s, \tau_R(x)) \).

**Theorem 4.5.** If \( f : S \rightarrow T \) and \( g : T \rightarrow E \) be a two nano contra \( \alpha \psi \) irresolute function, then their composition \( g \circ f \) is nano contra \( \alpha \psi \) irresolute function.

**Proof.** Let us take \( H \) be a nano \( \alpha \psi \) open set in \( E \). Then \( g^{-1}(H) \) is nano \( \alpha \psi \) closed in \( T \), because \( g \) is nano contra \( \alpha \psi \) irresolute. Now \( f^{-1}(g^{-1}(H)) \) is nano \( \alpha \psi \) open in \( S \), because \( f \) is nano contra \( \alpha \psi \) irresolute. Hence \( g \circ f \) is nano contra \( \alpha \psi \) irresolute function.

**Theorem 4.5.** Every nano contra \( \alpha \psi \) continuous function need not be nano contra \( \alpha \psi \) irresolute function, this is shown by the upcoming example.

**Example 4.6.** Let \( U = \{p, q, r, s\} \) with \( U/R = \{\{p, q\}, \{r, s\}\} \) and \( X = \{p, r\} \). Then \( \tau_R(X) = \{U, \phi, \{r\}, \{p, q, r\}, \{p, q\}\} \). Here the nano \( \alpha \psi \) open set is \( \tau_R^{\alpha \psi}(X) = P(U) \) and

\[ U = \{p, q, r, s\} \] with \( U/R = \{\{p\}, \{r\}, \{q, s\}\} \) and \( Y = \{p, q\} \). Then \( \tau_R(Y) = \{U, \phi, \{p\}, \{q, s\}, \{q, s\}\} \). Here the nano \( \alpha \psi \) open set is \( \tau_R^{\alpha \psi}(Y) = \{U, \phi, \{p\}, \{q, s\}, \{q, s\}\} \). Assume that \( f : (S, \tau_R(x)) \rightarrow (T, \tau_R(y)) \) be defined by \( f(p) = q, f(q) = r, f(r) = s, f(s) = p \). Then \( f^{-1}(p, q) = \{p, q\} \) is not nano \( \alpha \psi \) closed in \( (s, \tau_R(x)) \) for every nano \( \alpha \psi \) open set \( \{q, r\} \) in \( (T, \tau_R(y)) \).

## V. CONCLUSION

In this article, we introduced a notion of nano \( \alpha \psi \) kernal and nano contra \( \alpha \psi \) continuous function. Further we study some of their related attributes, theorems and results were discussed. Also we study the concept of nano contra \( \alpha \psi \) irresolute function and related theorems were discussed.

**REFERENCES**


